7.1 Integration by Parts

If u and v are differentiable functions of x then the product rule gives

$$\int \frac{d(uv)}{\cancel{dx}}\cancel{dx} = \int u \frac{dv}{\cancel{dx}}\cancel{dx} + \int v \frac{du}{\cancel{dx}}\cancel{dx}$$

$$uv = \int u\,dv + \int v\,du$$

$$\boxed{\int u\,dv = uv - \int v\,du}$$

L I A T E

Log, Inverse Trig, Algebraic, Trig, Exponential

P. 55 #3

$$I = \int \tan^{-1}(x)\,dx = x\tan^{-1}(x) - \int x \cdot \frac{1}{1+x^2}\,dx$$

$\tan^{-1}(x)$: I, dx: A

IP

$u = \tan^{-1}(x)$ $\quad dv = dx$

$du = \frac{1}{1+x^2}dx$ $\quad v = x$

Sub $\quad u = 1+x^2$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$$\int \frac{x}{1+x^2}\,dx = \int \frac{\frac{1}{2}du}{u}$$

$$= \frac{1}{2}\ln|u| + C = \frac{1}{2}\ln|1+x^2| + C$$

$$I = x\tan^{-1}(x) - \frac{1}{2}\ln|1+x^2| + C$$

7.1 Integration by parts $\int \underline{udv} = uv - \int vdu$

LIATE

1. $\int xe^{2x}\,dx = x\cdot\frac{1}{2}e^{2x} - \int \frac{1}{2}e^{2x}dx = \frac{x}{2}e^{2x} - \frac{1}{2}\cdot\frac{1}{2}e^{2x} + C$

   A E

   $u = x$, $dv = e^{2x}dx$

   $du = dx$, $v = \frac{1}{2}e^{2x}$

   $= \frac{e^{2x}}{2}\left(x - \frac{1}{2}\right) + C$

2. $\int x\ln(x)dx = \ln(x)\frac{x^2}{2} - \int \frac{x^2}{2}\cdot\frac{1}{x}dx = \frac{x^2}{2}\ln(x) - \frac{1}{2}\int x\,dx$

   A L

   $u = \ln(x)$, $dv = x\,dx$

   $du = \frac{1}{x}dx$, $v = \frac{x^2}{2}$

   $= \frac{x^2}{2}\ln(x) - \frac{1}{2}\cdot\frac{x^2}{2} + C$

   $= \frac{x^2}{2}\left(\ln(x) - \frac{1}{2}\right) + C$

One term:

3. $\int tan^{-1}(x)\,dx$

   I A

   $u = \tan^{-1}(x)$, $dv = dx$

   $du = \frac{1}{1+x^2}dx$, $v = x$

4. $\int \ln(x)\,dx$

IP

   $u = \ln(x)$, $dv = dx$

   $du = \frac{1}{x}dx$, $v = x$

   $\int \ln(x)dx = x\ln(x) - \int x\cdot\frac{1}{x}dx$

   $= x\ln(x) - x + C$

Repeated application

5. $I = \int x^2\cos(x)\,dx = x^2\sin(x) - 2\int x\sin(x)dx = x^2\sin(x) - 2\left(-x\cos(x) + \int\cos(x)dx\right)$

IP

   A T

   $u = x^2$, $dv = \cos(x)dx$

   $du = 2x\,dx$, $v = \sin(x)$

   $u = x$, $dv = \sin(x)dx$

   $du = dx$, $v = -\cos(x)$

   $I = x^2\sin(x) + 2x\cos(x) - 2\sin(x) + C$

6. $I = \int x^2e^{3x}dx = \frac{1}{3}x^2e^{3x} - \frac{2}{3}\int xe^{3x}dx$

IP

   $u = x^2$, $dv = e^{3x}dx$

   $du = 2x\,dx$, $v = \frac{1}{3}e^{3x}$

   $u = x$, $dv = e^{3x}dx$

   $du = dx$, $v = e^{3x}/3$

   $I = \frac{1}{3}x^2e^{3x} - \frac{2}{3}\left(\frac{1}{3}xe^{3x} - \frac{1}{3}\int e^{3x}dx\right)$

   $= \frac{1}{3}x^2e^{3x} - \frac{2}{9}xe^{3x} + \frac{2}{27}e^{3x} + C$

Reversal

7. $\int e^{2x}\cos(5x)\,dx$

 Understand the methods so you can solve similar problems.
Understand the concepts so you can solve unfamiliar problems.

Study the (a) class notes, (b) do the 4th hour problems, (c) study the text examples, (d) do the text exercises and (e) read the next text section.

P. 55 #7, $I = \int e^{2x}\cos(5x)\,dx = \frac{1}{2}e^{2x}\cos(5x) + \frac{5}{2}\int e^{2x}\sin(5x)\,dx$

IP

$u = \cos(5x) \quad dv = e^{2x}dx$
$du = -\sin(5x)5 \quad v = \frac{1}{2}e^{2x}$

$u = \sin(5x) \quad dv = e^{2x}dx$
$du = \cos(5x)5 \quad v = \frac{1}{2}e^{2x}$

$$I = \frac{1}{2}e^{2x}\cos(5x) + \frac{5}{2}\left\{\frac{1}{2}e^{2x}\sin(5x) - \frac{5}{2}\int e^{2x}\cos(5x)\,dx\right\}$$

$$I = \frac{1}{2}e^{2x}\cos(5x) + \frac{5}{4}e^{2x}\sin(5x) - \frac{25}{4}I$$

$$\frac{4}{4}I + \frac{25}{4}I = \frac{29}{4}I =$$

$$I = \frac{4}{29}\left(\frac{1}{2}e^{2x}\cos(5x) + \frac{5}{4}e^{2x}\sin(5x)\right) + C$$

P. 56 #10 $\quad \int_1^e \sin(\ln(x))\,dx$

$$I = \int \sin(\ln(x))\,dx = x\sin(\ln(x)) - \int \cancel{x}\cos(\ln(x))\frac{1}{\cancel{x}}\,dx$$

$u = \sin(\ln(x)) \quad dv = dx$
$du = \cos(\ln(x))\frac{1}{x}dx \quad v = x$

$u = \cos(\ln(x)) \quad dv = dx$
$du = -\sin(\ln(x))\frac{1}{x}dx \quad v = x$

$$I = x\sin(\ln(x)) - \left(x\cos(\ln(x)) - \int -\sin(\ln(x))\frac{1}{\cancel{x}}\cancel{x}\,dx\right)$$

$$I = x\sin(\ln(x)) - x\cos(\ln(x)) - I$$

$$2I =$$

$$I = \frac{x}{2}\left(\sin(\ln(x)) - \cos(\ln(x))\right) + C$$

$$\int_1^e \sin(\ln(x))\,dx = \left[\frac{x}{2}\left(\sin(\ln(x)) - \cos(\ln(x))\right)\right]_1^e$$
$$= \frac{e}{2}\left(\sin(1) - \cos(1)\right) - \frac{1}{2}(-1) = \frac{e}{2}(\sin 1 - \cos 1) + \frac{1}{2}$$

Definite Integral: 8. $\int_{-1/2}^{1/2} sin^{-1}(x)dx$ $= 0$

9. $\int_{-1/2}^{0} sin^{-1}(x)dx$

IP $\int \sin^{-1}(x)\,dx = x\sin^{-1}(x) - \int \frac{x}{\sqrt{1-x^2}}\,dx$

$u = \sin^{-1}(x)$ $\quad dv = dx$

$du = \frac{1}{\sqrt{1-x^2}}\,dx$ $\quad v = x$

Sub $u = 1-x^2$

$du = -2x\,dx$

$-\frac{1}{2}du = x\,dx$

$\int \frac{x}{\sqrt{1-x^2}} = -\frac{1}{2}\int \frac{du}{\sqrt{u}}$

$= -\frac{1}{2}\int u^{-1/2}du$

$= -\frac{1}{2}\frac{u^{1/2}}{1/2} + C$

$= -\sqrt{1-x^2} + C$

$\int \sin^{-1}(x)\,dx = x\sin^{-1}(x) + \sqrt{1-x^2} + C$

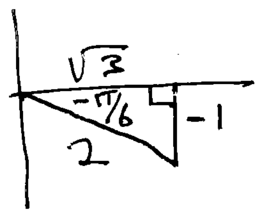
$= \left[x\sin^{-1}(x) + \sqrt{1-x^2}\right]_{-1/2}^{0}$

$= 1 - \left(-\frac{1}{2}\sin^{-1}\left(-\frac{1}{2}\right) + \sqrt{\frac{3}{4}}\right)$

~~10.~~ $\cancel{\int_1^e \sin(\ln(x))\,dx}$

$= 1 - \left(-\frac{1}{2}\left(-\frac{\pi}{6}\right) + \frac{\sqrt{3}}{2}\right)$

$= 1 - \frac{\pi}{12} - \frac{\sqrt{3}}{2}$



Combining u-substitution and IP

11. $\int x^3\, sin(x^2)dx$ $= \int u \sin(u)\frac{1}{2}du$

Sub $u = x^2$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$ | IP

12. $\int e^{x^2} x^3 dx$ $= \int e^{w} w \frac{1}{2}dw$

Sub $w = x^2$

$dw = 2x\,dx$

$\frac{1}{2}dw = x\,dx$ | IP

Omit reduction formulas. p.402 and example 7.

 Understand the methods so you can solve similar problems.
Understand the concepts so you can solve unfamiliar problems.

Study the (a) class notes, (b) do the 4th hour problems, (c) study the text examples, (d) do the text exercises and (e) read the next text section.