6.5

Example A force of f(x) = $\sqrt{2+x}$ lb is applied in moving an object from x = 2 to x = 14 ft. How much work is performed?

$$W = \int_2^{14} \sqrt{2+x}\,dx = \frac{2}{3}(2+x)^{3/2}\Big|_2^{14}$$

$$= \frac{112}{3} \text{ ft-lb}$$

Hooke's Law says the force F required to maintain a spring x units *beyond its natural length* is F = kx, where k is the positive *spring constant.*

Example A force of 15 N is required to maintain a spring stretched from its natural length of 5 cm to 8 cm. How much work is done in stretching the spring from 8 to 12 cm?

.05m .08m .08, .12m

Solution: 1. Find k $F = kx$

$15 = k(.03)$, $k = \frac{15}{.03} = 500$

$.08 - .05$, $x = .03$ m

2. So F(x) = kx = $500x$ N

3. Find W = $\int_a^b f(x)dx$ =

$.08 - .05 = .03$ $\quad .12 - .05 = .07$

$$= \int_{.03}^{.07} 500x\,dx = 1\text{J}$$

Study the (a) class notes, (b) do the 4th hour problems, (c) study the text examples, (d) do the text exercises and (e) read the next text section.

PUMPING FLUIDS

In calculating work in metric units, a useful method follows:

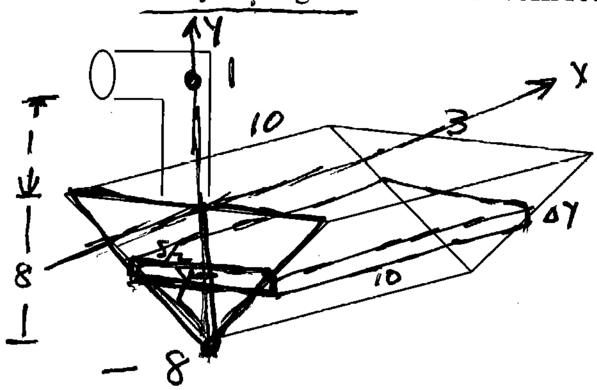
Use these equations:

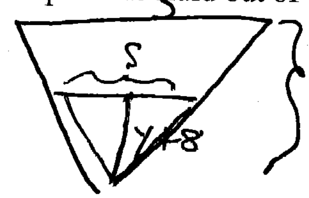
Mass m = (density) volume

Force f = (mass) gravity (and use 9.8 for the gravitational constant)

Work w = (force) distance

Example: A tank in the shape of a cylinder of length 10 m whose vertical cross sections are isosceles triangles of height 8 m and base 3 m is buried 1 m below ground and filled with a fluid of density 250 kg/m^3. Find the work required to pump all the fluid out of the tank.





$y-(-8) = y+8$

By similar Δs

$\frac{s}{y+8} = \frac{3}{8}$

$s = \frac{3}{8}(y+8)$

Solution: Here we have a variable volume, and a variable distance.

1. Calculate V_i for a horizontal slice of height Δy, length 10 m, and width dependent on y.

$V_i = s\,10\,\Delta y = \frac{3}{8}(y+8)\,10\,\Delta y = \frac{15}{4}(y+8)\Delta y \text{ m}^3$

2. Then the mass for this slice is $m_i = V_i$ (density)

$m_i = 250 \cdot \frac{15}{4}(y+8)\Delta y$

3. The force $f_i = m_i\, g$

$f_i = 9.8\,(250)\frac{15}{4}(y+8)\Delta y$

4. The work $W_i = f_i\, d_i$ where the distance d_i depends on the height of the slab.

$d_i = 1-y, \quad W_i = 9.8\,(250)\frac{15}{4}(y+8)(1-y)\Delta y$

5. Now W = $\lim\left(\sum W_i\right)$ which can be evaluated as a definite integral.

$$W = \int_{-8}^{0} 9.8\,(250)\,\frac{15}{4}(y+8)(1-y)\,dy = 1078{,}000 \text{ J}$$

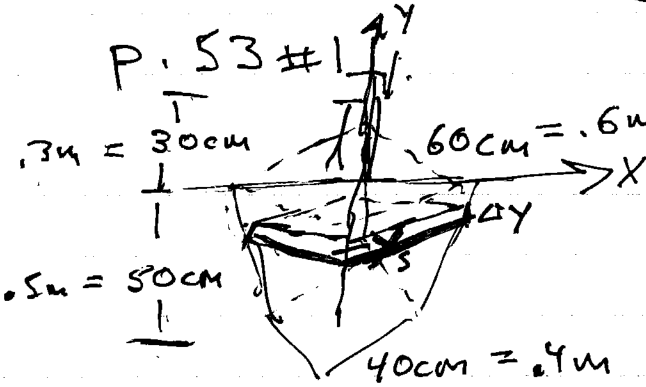
or 1078 kJ

g density volume distance

Study the (a) class notes, (b) do the 4th hour problems, (c) study the text examples, (d) do the text exercises and (e) read the next text section.

6.5 Work

P. 53 #1

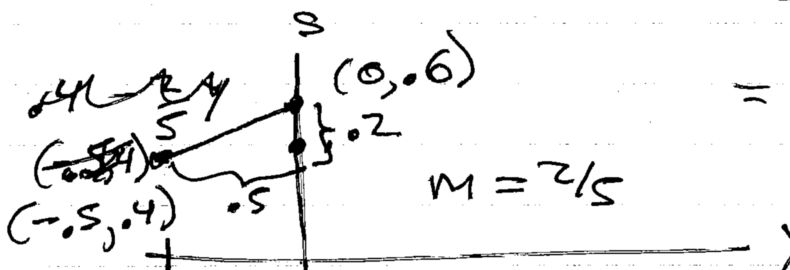


density $= 1.5g/cm^3$

$= 1.5 \frac{\cancel{gm}}{\cancel{cm^3}} \frac{kg}{1000\cancel{gm}} \frac{(100\cancel{cm})^3}{m^3}$

$= \frac{1.5 \cdot 100^3}{1000} kg/m^3$

$= 1500\ kg/m^3$



$m = 2/5$

$s - .6 = \frac{2}{5}(y - 0)$

$s = \frac{2}{5}y + .6$

$A = s^2 = (\frac{2}{5}y + .6)^2$

$V = (\frac{2}{5}y + .6)^2 \Delta y$

$a = 9.8$

density $= 1500\ kg/m^3$

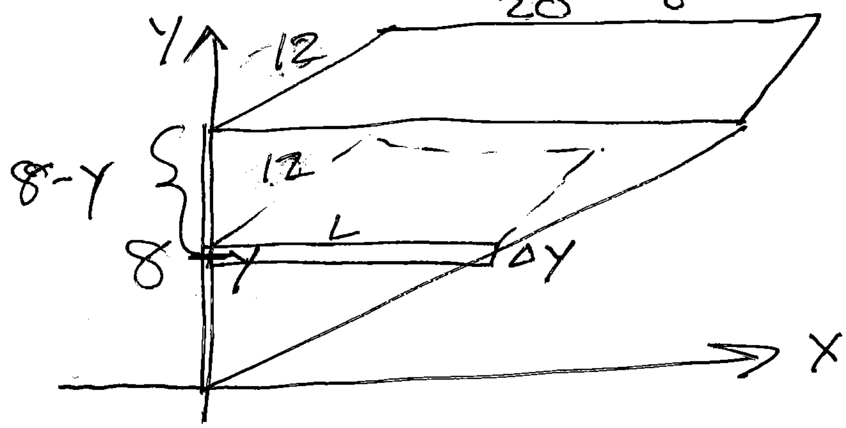
distance $= .3 - y$

$$W = \int_{-.5}^{0} 9.8(1500)(\tfrac{2}{5}y + .6)^2(.3 - y)\,dy$$

a density volume distance

(2

P.53 #2 $W = 62.4\int_0^8 30y(8-y)\,dy$



$\frac{L}{y} = \frac{20}{8} = \frac{10}{4}$

$L = \frac{10}{4}y$

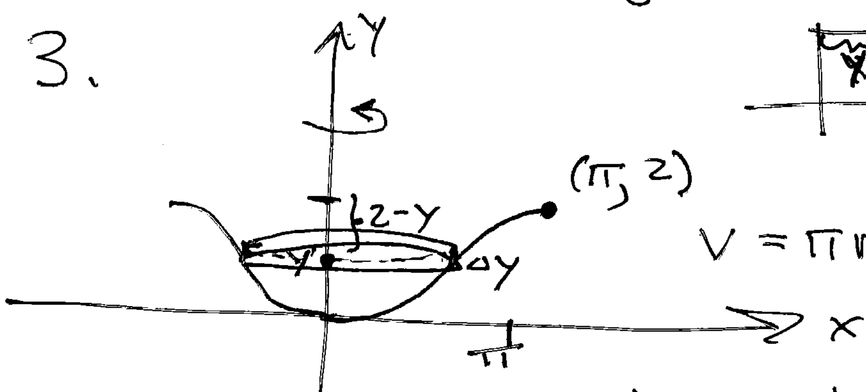
$V = \frac{10}{4}y \cdot 12\,\Delta y = 30y\,\Delta y$

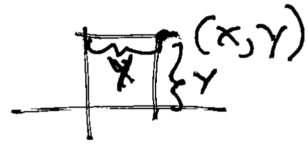
$D = 8-y$

Force $= 62.4$ lb/ft^3

$W = 62.4\int_0^8 30y(8-y)\,dy$

3.





$V = \pi r^2 h = \pi r^2 \Delta y = \pi x^2 \Delta y$

$V = \pi\left(\frac{\pi}{2} + \sin^{-1}(y-1)\right)^2 \Delta y$

$y = 1 + \sin\left(x - \frac{\pi}{2}\right)$

$\sin^{-1}(y-1) = x - \pi/2$, $x = \frac{\pi}{2} + \sin^{-1}(y-1)$

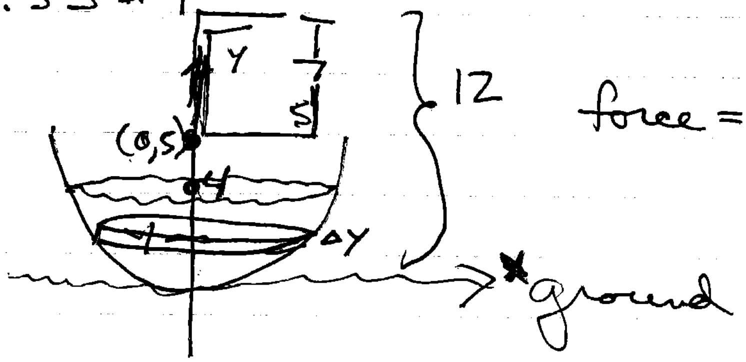
distance $= 2-y$

$a = 9.8$

density $= \rho$

$W = 9.8\rho\int_0^2 \pi\left(\frac{\pi}{2} + \sin^{-1}(y-1)\right)^2 (2-y)\,dy$

P.53 #4



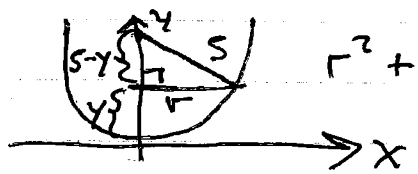
force = 55 lb

$(x-a)^2 + (y-b)^2 = r^2$

$x^2 + (y-5)^2 = 5^2$

$x^2 = 25 - (y^2 - 10y + 25)$

$= 10y - y^2$



$r^2 + (5-y)^2 = 5^2$

$V = \pi r^2 h$

$= \pi r^2 \Delta y$

$= \pi x^2 \Delta y$

$= \pi (10y - y^2) \Delta y \ ft^3$

~~force~~ weight $= 55 \ lb/ft^3$

force $= wt \cdot V = 55\pi(10y - y^2)\Delta y$ lb

distance $= 12 - y$

$$W = \int_0^4 55\pi(10y - y^2)(12-y)\,dy$$