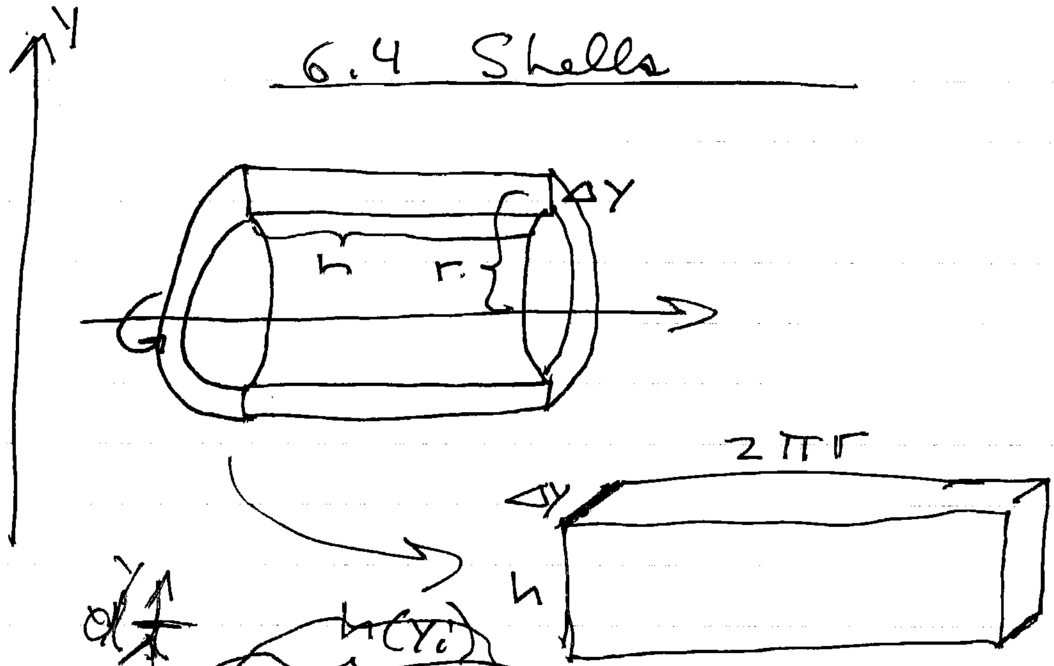
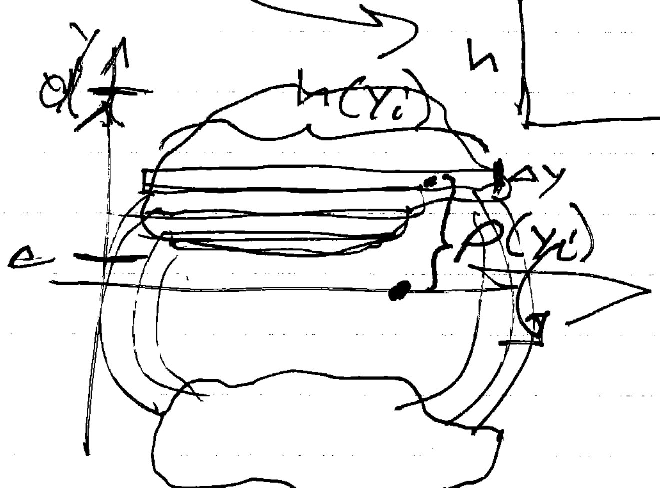
6.4 Shells



$V \approx 2\pi r h \Delta y$



partition $[c, d]$

$c = y_0 < y_1 < \cdots < y_n = d$

$\Delta y = \frac{d-c}{n}$, $y_i = a + i\Delta y$

$$V \approx \sum_{i=1}^{n} 2\pi \rho(y_i) h(y_i) \Delta y$$

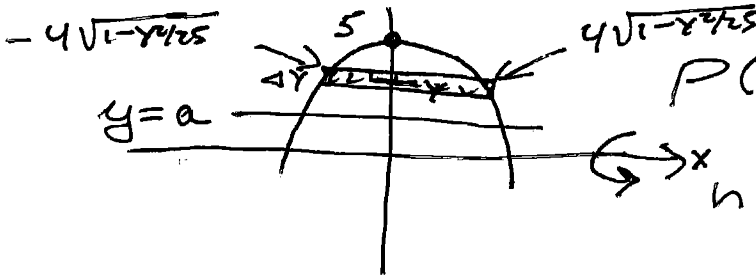
If h is continuous, take $\lim_{n \to \infty} V$

$$V = \int_c^d 2\pi \rho(y) h(y)\, dy$$

$$\text{or } \underline{2\pi} \int_c^d \rho(y) h(y)\, dy$$

cut is parallel

P. 48, 3(a) Shells, parallel, 2π



$P(y) = y$

$h(y) = 4\sqrt{1-y^2/25} - (-4\sqrt{1-y^2/25})$

$= 8\sqrt{1-y^2/25}$

$x = \pm 4\sqrt{1-y^2/25}$

$$V = 2\pi \int_a^5 y\, 8\sqrt{1-y^2/25}\, dy$$

6.4 Volume by cylindrical shells

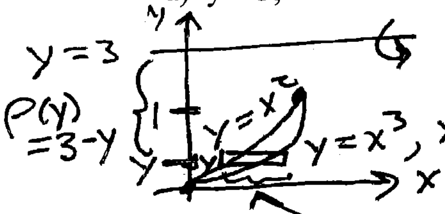
If a region is rotated about a vertical line to create a solid S, the volume can be calculated by taking a cut parallel to the line, of height $h(x)$ and at distance $p(x)$ from the line. Rotating this cut about the axis of revolution yields a cylindrical shell. In this case the volume is

$$V(S) = 2\pi \int_a^b p(x)h(x)dx$$

1. Use cylindrical shells to set up an integral for the volume of the solid obtained by rotating the region bounded by $y = x^3$ and $y = x^2$, $0 \le x \le 1$ about the following:

a) y = 3;

Shells parallel, 2π

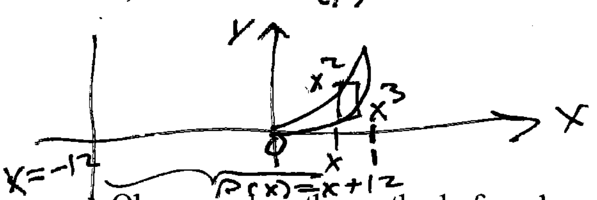


$$V = 2\pi \int_0^1 (3-y)(\sqrt[3]{y} - \sqrt{y})\,dy$$

$h(y) = \sqrt[3]{y} - \sqrt{y}$

b) x = -12.

$$V = 2\pi \int_0^1 (x+12)(x^2 - x^3)\,dx$$



Observe when the method of washers is best, and when the method of shells is best:

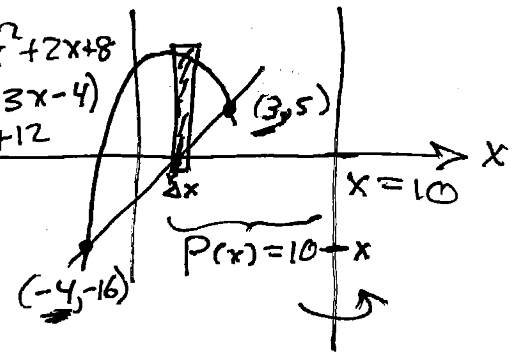
2. Set up an integral expression for the exact volume of the solid S obtained by rotating the region R bounded by $f(x) = 3x - 4$ and $g(x) = -x^2 + 2x + 8$ about the line

(a) x = 10;

Shells parallel, 2π

$h(x) = -x^2 + 2x + 8 - (3x-4)$
$= -x^2 - x + 12$

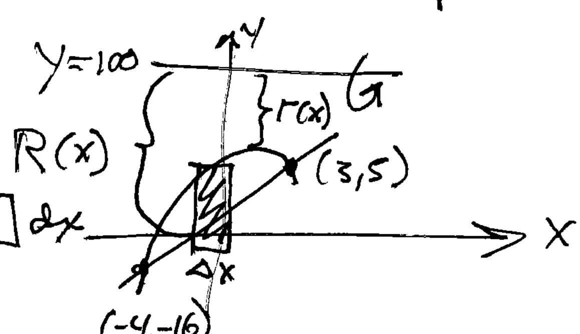
$$V = 2\pi \int_{-4}^{3} (10-x)(12 - x^2 - x)\,dx$$



(b) y = 100. Washers $\perp$, π

$R(x) = 100 - (3x-4)$
$= 104 - 3x$
$r(x) = 100 - (-x^2 + 2x + 8)$
$= 92 + x^2 - 2x$

$$V = \pi \int_{-4}^{3} \left[(104-3x)^2 - (92 + x^2 - 2x)^2 \right] dx$$



 Understand the methods so you can solve similar problems.
Understand the concepts so you can solve unfamiliar problems.

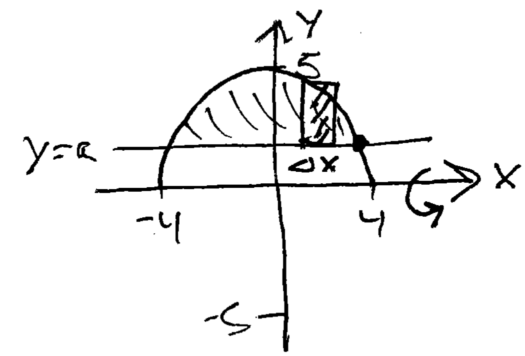
Study the (a) class notes, (b) do the 4th hour problems, (c) study the text examples, (d) do the text exercises and (e) read the next text section.

3. Set up an integral expression for the exact volume of the solid S obtained by rotating the region R bounded by the ellipse $\frac{x^2}{16}+\frac{y^2}{25}=1$ and above the line $y=a, 0<a<5$

(a) about the x-axis;

Washers $x=\pm 4\sqrt{1-y^2/25}$

when $y=a$, $x=\pm 4\sqrt{1-a^2/25}$



$y=\pm 5\sqrt{1-x^2/16}$

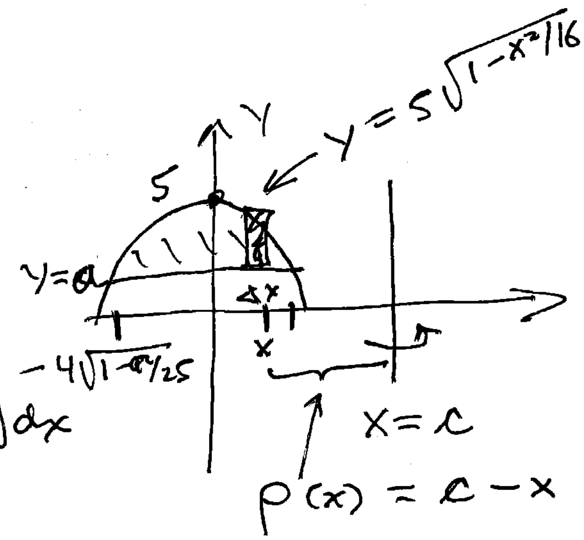
$R(x)=5\sqrt{1-x^2/16}$

$r(x)=a$

$$V=\pi\int_{-4\sqrt{1-a^2/25}}^{4\sqrt{1-a^2/25}}\left(25\left(1-x^2/16\right)-a^2\right)dx$$

(b) about the line $x=c$, $c>5$.

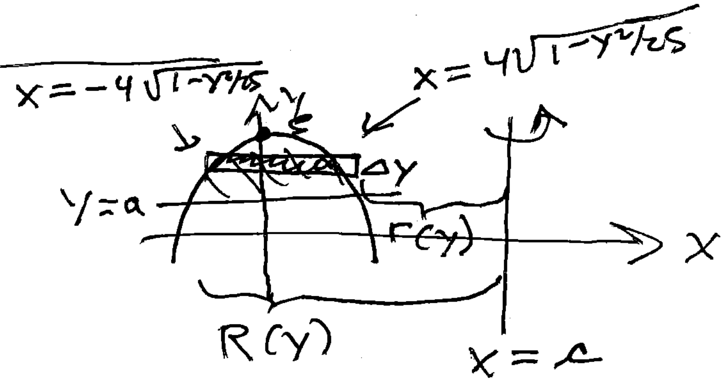
Shells, parallel, 2π



$h(x)=5\sqrt{1-x^2/16}-a$

$$V=2\pi\int_{-4\sqrt{1-a^2/25}}^{4\sqrt{1-a^2/25}}(c-x)\left(5\sqrt{1-x^2/16}-a\right)dx$$

Washers, $\perp$, π



$R(y)=c-\left(-4\sqrt{1-y^2/25}\right)$

$=c+4\sqrt{1-y^2/25}$

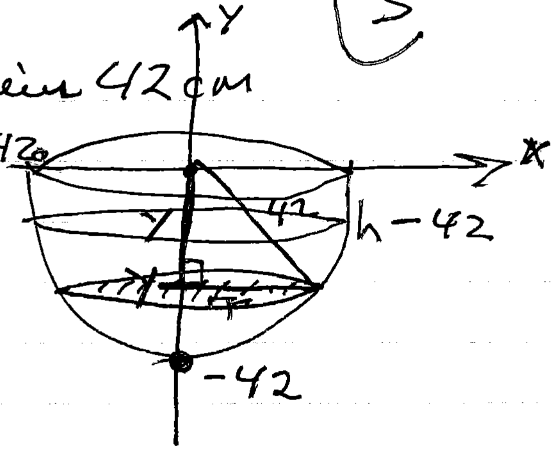
$r(y)=c-4\sqrt{1-y^2/25}$

$$V=\pi\int_a^5\left[\left(c+4\sqrt{1-y^2/25}\right)^2-\left(c-4\sqrt{1-y^2/25}\right)^2\right]dy$$

Understand the concepts so you can solve unfamiliar problems.

Study the (a) class notes, (b) do the 4th hour problems, (c) study the text examples, (d) do the text exercises and (e) read the next text section.

4. A hemispherical bowl of radius 42 cm is filled to height h cm, $0 < h < 42$. Set up an integral expression for the volume of water in the bowl.



Known CSA $\quad V = \int_c^d A(y)\,dy$

$A = \pi r^2 \qquad y^2 + r^2 = 42^2, \quad r^2 = 42^2 - y^2$

$A = \pi(42^2 - y^2), \quad V = \int_{-42}^{h-42} \pi(42^2 - y^2)\,dy$

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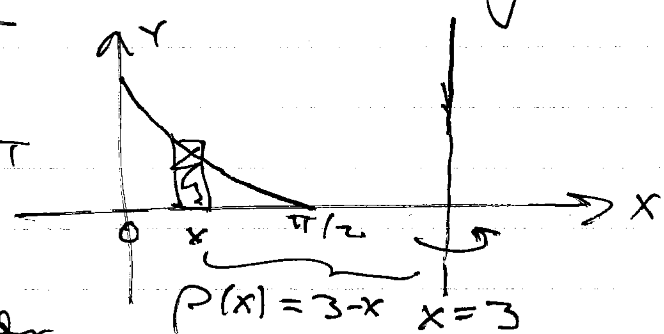
5. The region R is bounded by $f(x) = 1 - \sin(x)$, $0 \le x \le \pi/2$, and the coordinate axes. Set up an integral expression for the volume of the solid obtained by rotating R about

(a) $x = 3$

Shells parallel, 2π

$h(x) = 1 - \sin(x)$

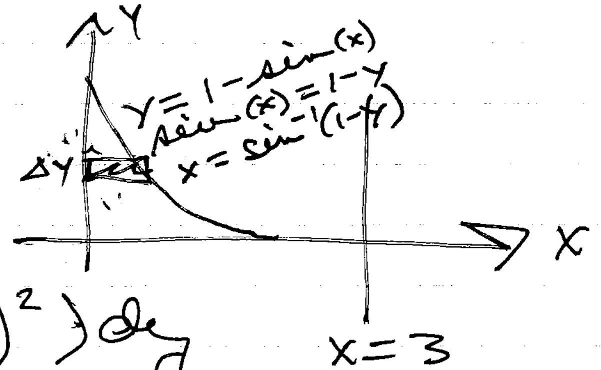


$V = 2\pi \int_0^{\pi/2} (3 - x)(1 - \sin(x))\,dx$

Washers

$R(y) = 3$

$r(y) = 3 - \sin^{-1}(1 - y)$



$V = \pi \int_0^1 \left(9 - (3 - \sin^{-1}(1 - y))^2\right) dy$