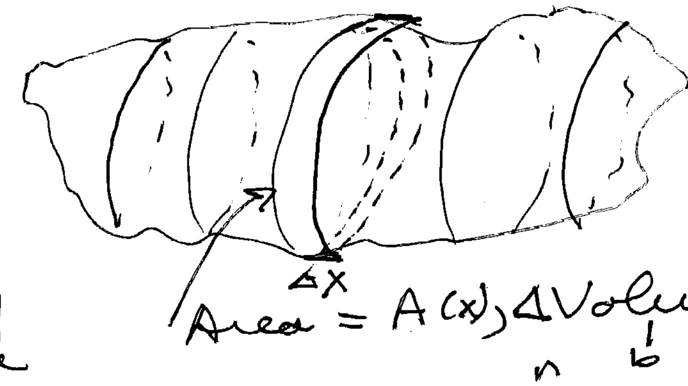
<u>6.2 Volume</u>



Area $= A(x)$, ΔVolume $= A(x)\Delta x$

so Volume $\cong \sum_{i=1}^{n} A_i(x)\Delta x$

and $V = \int_a^b A(x)\,dx$

<u>volume of known cross-sectional area</u>

6.2Volume and Average Value

In this section of the text omit Density on pp 367 to 369.

Volumes of known cross sectional area

If a solid S has a base region Adefined for x in $[a, b]$and the cross-sections of the solid taken perpendicular to the *x-axis* are $A(x)$, then small portions of the solid have volume approximately $A(x_i)\Delta x$ after partitioning the *x-axis* in the usual way. Consequently the volume of the solid can be approximated as

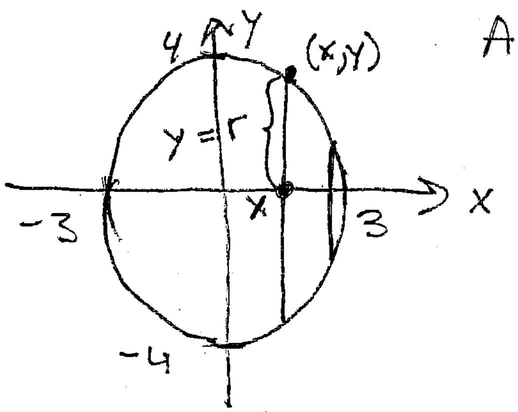
$$V(S) \approx \sum_{i=1}^{n} A(x_i)\Delta x$$

and at the limit the volume is

$$V(S) = \int_a^b A(x)dx$$

A similar formula applies if the cross-sections are parallel to the *y-axis*.

1. Set up an integral for the volume of a solid whose base is the ellipse $\frac{x^2}{9} + \frac{y^2}{16} = 1$ and whose cross-sections perpendicular to the *x-axis* are semicircles.



$$A_{semicircle} = \frac{1}{2}\pi r^2$$

$$\frac{y^2}{16} = 1 - \frac{x^2}{9}$$

$$\frac{y}{4} = \pm\sqrt{1 - x^2/9}$$

$$y = \pm 4\sqrt{1 - x^2/9}$$

$$r = 4\sqrt{1 - x^2/9}$$

$$A_{semicircle} = \frac{1}{2}\pi r^2$$

$$= \frac{\pi}{2}\left(16\left(1 - \frac{x^2}{9}\right)\right)$$

$$= 8\pi\left(1 - \frac{x^2}{9}\right)$$

$$V = \int_a^b A(x)dx = \int_{-3}^{3} 8\pi\left(1 - \frac{x^2}{9}\right) dx$$

 Understand the methods so you can solve similar problems.
Understand the concepts so you can solve unfamiliar problems.

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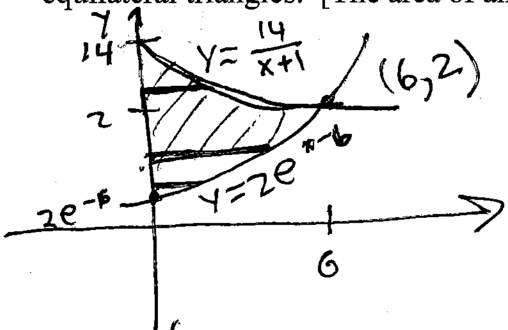
Study the (a) class notes, (b) do the 4th hour problems, (c) study the text examples, (d) do the text exercises and (e) read the next text section.

2. Set up an integral for the volume of a solid whose base is the region *A* bounded by the *y-axis*, $y = 2e^{x-6}$, $y = \dfrac{14}{x+1}$ and whose cross-sections perpendicular to the *y-axis* are equilateral triangles. [The area of an equilateral triangle of side *s* is $= \frac{s^2\sqrt{3}}{4}$.]



$x+1 = \frac{14}{y}$

$x = \frac{14}{y} - 1$

$A(y) = \left(\frac{14}{y} - 1\right)^2 \sqrt{3}/4$

$e^{x-6} = y/2$

$x - 6 = \ln(y/2)$

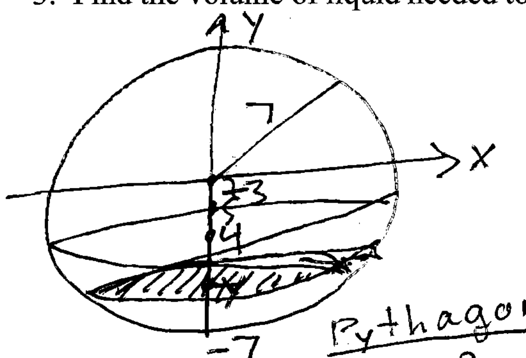
$x = 6 + \ln\left(\frac{y}{2}\right)$

$A(y) = \left(6 + \ln\left(\frac{y}{2}\right)\right)^2 \sqrt{3}/4$

$$\text{so } V = \int_{2e^{-6}}^{2} \left(6 + \ln\left(\frac{y}{2}\right)\right)^2 \frac{\sqrt{3}}{4}\,dy + \int_{2}^{14} \left(\frac{14}{y} - 1\right)^2 \frac{\sqrt{3}}{4}\,dy$$

For the following, provide an appropriate sketch, set up the integral for the exact value, and approximate to 3 decimal accuracy using fnInt.

3. Find the volume of liquid needed to fill a sphere of radius 7 m to a height of 4 m.

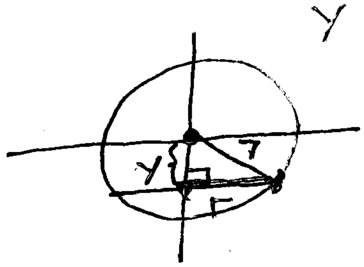


$A = \pi r^2$

Pythagorus

$y^2 + r^2 = 7^2$

$r^2 = 49 - y^2$



so $A = \pi r^2$

$= \pi(49 - y^2)$

$$V = \int_{-7}^{-3} \pi(49 - y^2)\,dy = \pi\, 90.667 \text{ m}^3$$

or 284.838 m^3

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Average Value

$$\text{mean} = \text{average value} = \frac{x_1 + x_2 + \cdots + x_n}{n} = \overline{X}$$

$$\overline{X} = \frac{\sum_{i=1}^{n} x_i}{n}$$

Suppose f is continuous on $[a,b]$, $\Delta x = \frac{b-a}{n}$

Let $x_i = a + i\Delta x$ for $i = 0, 1, 2, \ldots, n$

The average of $f(x_1), f(x_2), \ldots, f(x_n)$ is

$$AVG = \frac{\sum_{i=1}^{n} f(x_i)}{n}$$

$$n = \frac{b-a}{\Delta x}$$

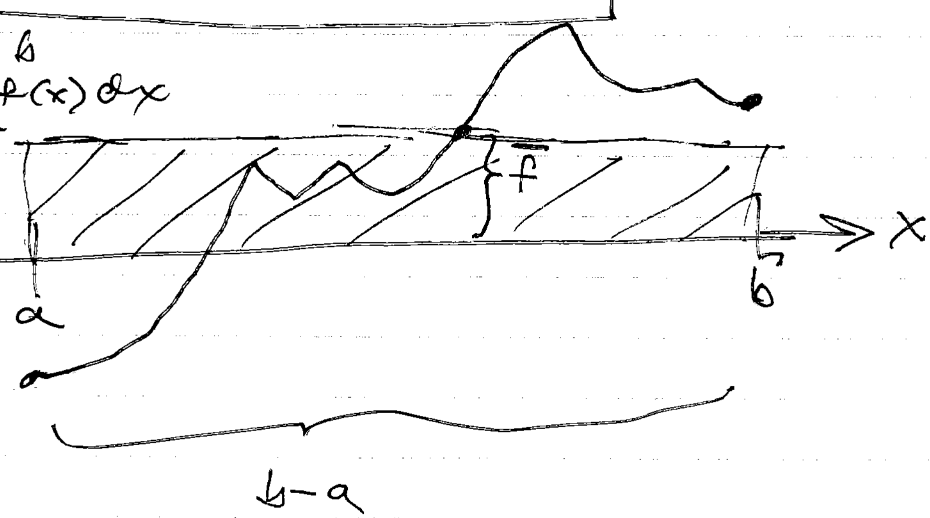
$$= \frac{\sum f(x_i)}{\frac{b-a}{\Delta x}}$$

$$= \frac{1}{b-a} \sum f(x_i) \Delta x$$

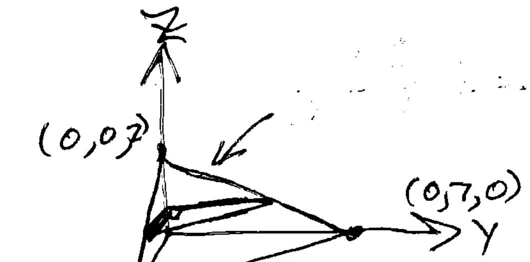
As $n \to \infty$, $\|\Delta x\| \to 0$ so the average value of f on $[a,b]$ is

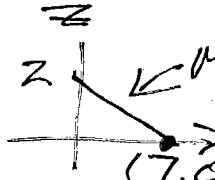
$$\boxed{\overline{f} = \frac{1}{b-a} \int_a^b f(x)\, dx}$$

$$(b-a)\overline{f} = \int_a^b f(x)\, dx$$



4. Find the volume of the solid bounded by the 3 coordinate planes and the plane through the points (3,0,0), (0,7,0) and (0,0,2).



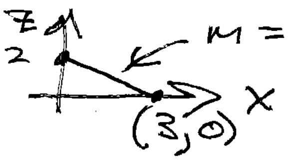


$z - z_1 = m(y - y_1)$

$z - 0 = -\frac{2}{7}(y-7)$

$z = -\frac{2}{7}(y-7)$

$-\frac{7}{2}z + 7 = y$



$z - 0 = -\frac{2}{3}(x-3)$

$-\frac{3}{2}z + 3 = x$

Average value= $\bar{f} = \frac{1}{b-a}\int_a^b f(x)dx$

$A(z) = \frac{1}{2}xy$

$= \frac{1}{2}(7-\frac{7}{2}z)(3-\frac{3}{2}z)$

$V = \int_0^2 \frac{1}{2}(7-\frac{7}{2}z)(3-\frac{3}{2}z)dz$

$= 7$

5. Find the average value of $f(x) = \sqrt{1+x}$ on [0, 8].

$\bar{f} = \frac{1}{8-0}\int_0^8 \sqrt{1+x}\,dx$

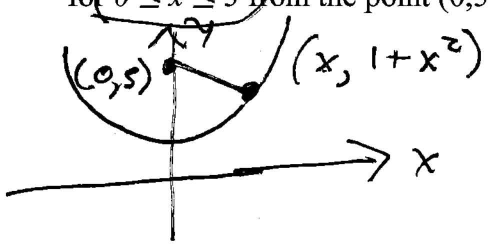
$= \frac{1}{\cancel{8}\,4}\,\frac{\cancel{2}}{3}(1+x)^{3/2}\Big|_0^8$

$= \frac{1}{12}[9^{3/2} - 1^{3/2}] = \frac{1}{12}(27-1) = \frac{26}{12} = \frac{13}{6}$

6. Find the average value of the distance of all points on the curve

$f(x) = 1 + x^2$

for $0 \le x \le 3$ from the point (0,5). Give your answer to 3 decimal place accuracy.



$(x, 1+x^2)$ $\quad D = \sqrt{(x_1-x_2)^2 + (y_1-y_2)^2}$

$= \sqrt{x^2 + (1+x^2-5)^2}$

$= \sqrt{x^2 + (x^2-4)^2}$

$\bar{f} = \frac{1}{3-0}\int_0^3 \sqrt{x^2+(x^2-4)^2}\,dx$

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