5.7

$$\frac{d}{dx}\sin^{-1}(x) = \frac{1}{\sqrt{1-x^2}} \text{ so } \int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1}(x) + C$$

$$\frac{d}{dx}\tan^{-1}(x) = \frac{1}{1+x^2} \text{ so } \int \frac{dx}{1+x^2}\,dx = \tan^{-1}(x) + C$$

$$\frac{d}{dx}\ln|x| = \frac{1}{x} \text{ so } \int \frac{1}{x}\,dx = \ln|x| + C$$

$$\frac{d}{dx}a^x = a^x\ln(a) \text{ } \cancel{\text{so}} \int a^x\,dx = \frac{a^x}{\ln(a)} + C \text{ , } \begin{matrix} a>0 \\ a\neq 1 \end{matrix}$$

5.7 Further Transcendental Functions

$$\int \frac{dx}{\sqrt{1-x^2}} = sin^{-1}(x) + C, \quad \int \frac{dx}{1+x^2} = tan^{-1}(x) + C, \quad \int \frac{dx}{x} = \ln|x| + C$$

$$\int a^x dx = \frac{a^x}{\ln(a)} + C$$

We omit arcsecant, so omit Example 2.

1. $\int_0^6 \frac{dx}{9x^2+25}$

$$= \frac{1}{25}\int_0^6 \frac{dx}{\frac{9x^2}{25}+1} = \frac{1}{25}\int_0^{18/5} \frac{\frac{5}{3}du}{u^2+1} = \frac{1}{15}\tan^{-1}(u)\Big|_0^{18/5}$$

Let $u = \frac{3x}{5}$

x	u
0	0
6	18/5

$du = \frac{3}{5}dx$, $\frac{5}{3}du = dx$

$$= \frac{1}{15}\left(\tan^{-1}\left(\frac{18}{5}\right) - \underbrace{\tan^{-1}(0)}_{=0}\right)$$

$$= \frac{1}{15}\tan^{-1}\left(\frac{18}{5}\right)$$

2. $\int \frac{dt}{\sqrt{5-t^2}} = \frac{1}{\sqrt{5}}\int \frac{dt}{\sqrt{1-(t/\sqrt{5})^2}} = \frac{1}{\sqrt{5}}\int \frac{\sqrt{5}\,du}{\sqrt{1-u^2}}$

$\sqrt{5-t^2}$

$= \sqrt{5-5t^2/5}$

$= \sqrt{5(1-t^2/5)}$

$= \sqrt{5}\sqrt{1-(t/\sqrt{5})^2}$

Let $u = t/\sqrt{5}$

$du = \frac{1}{\sqrt{5}}dt$

$\sqrt{5}\,du = dt$

$= \sin^{-1}(u) + C$

$= \sin^{-1}\left(\frac{t}{\sqrt{5}}\right) + C$

3. $\int \frac{3x}{x^4+1}dx = 3\int \frac{\frac{1}{2}du}{u^2+1} = \frac{3}{2}\tan^{-1}(u) + C$

Let $u = x^2$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$= \frac{3}{2}\tan^{-1}(x^2) + C$

$m^x n^x = (mn)^x$

$\int a^x dx = \frac{a^x}{\ln(a)} + C$

$= \frac{(2e)^7 - (2e)^{-3}}{\ln(2e)}$

4. $\int_{-3}^{7} 2^x e^x dx$

 Understand the methods so you can solve similar problems.
Understand the concepts so you can solve unfamiliar problems.

Study the (a) class notes, (b) do the 4th hour problems, (c) study the text examples, (d) do the text exercises and (e) read the next text section.

5. $\int \frac{dx}{\sqrt{16-3x^2}}$ [Factor 16. Substitute $u = \frac{\sqrt{3}}{4}x$.Use $\frac{d}{du}(sin^{-1}(u)) = \frac{1}{\sqrt{1-u^2}}$.]

$= \int \frac{dx}{\sqrt{16 - \frac{16 \cdot 3x^2}{16}}}$ Substitute $u = \frac{x\sqrt{3}}{4}$

$du = \frac{\sqrt{3}}{4} dx$

$= \int \frac{dx}{\sqrt{16}\sqrt{1-\frac{3x^2}{16}}}$ $\frac{4}{\sqrt{3}} du = dx$

$= \frac{1}{4}\int \frac{dx}{\sqrt{1-\left(\frac{x\sqrt{3}}{4}\right)^2}} = \frac{1}{4}\int \frac{\frac{4}{\sqrt{3}} du}{\sqrt{1-u^2}} = \frac{1}{\sqrt{3}} \sin^{-1}(u) + C = \underline{\frac{1}{\sqrt{3}} \sin^{-1}\left(\frac{x\sqrt{3}}{4}\right) + C}$

6.. $\int \frac{e^{\cos(x)}}{\csc(x)} dx = \int e^{\cos(x)} \sin(x)\, dx = -\int e^u du = -e^u + C$

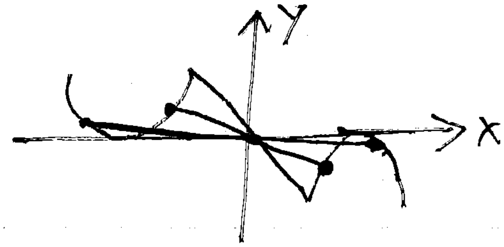
$\underline{= -e^{\cos(x)} + C}$ $u = \cos(x)$
$du = -\sin(x) dx$

 Understand the methods so you can solve similar problems.
Understand the concepts so you can solve unfamiliar problems.

Study the (a) class notes, (b) do the 4th hour problems, (c) study the text examples, (d) do the text exercises and (e) read the next text section.

5.7 Odd & Even Functions

the function $f(x)$ is ODD if $f(-x) = -f(x)$
is a refection over the origin.



If f is odd then $\int_{-a}^{a} f(x)\,dx = 0$

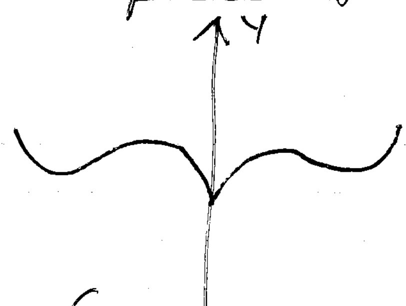
Ex. $\int_{-10}^{10} \sin(x^3)\,dx$

$$f(x) = \sin(x^3)$$
$$f(-x) = \sin((-x))^3 = \sin(-x^3)$$
$$= -\sin(x^3) = -f(x)$$

so f is odd

so $\int_{-10}^{10} \sin(x^3) = 0$

The function f is EVEN if $f(-x) = f(x)$
is a refection through the y-axis.



and if f is even
$$\int_{-a}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx$$

Ex. $\int_{-6}^{6} |x|\,dx = 2\int_{0}^{6} |x|\,dx = 2\int_{0}^{6} x\,dx = \cancel{2}\frac{x^2}{\cancel{2}}\Big|_0^6 = 36$

$|x|$ is even

$$\int_{-\pi}^{\pi} \frac{x^3+x}{x^2+\cos(x)}\,dx$$

$$f(-x) = \begin{cases} f(x) & \text{EVEN} \\ -f(x) & \text{ODD} \\ \text{otherwise} & \text{NEITHER} \end{cases}$$

$$f(x) = \frac{x^3+x}{x^2+\cos(x)}$$

$$f(-x) = \frac{(-x)^3+(-x)}{(-x)^2+\cos(-x)}$$

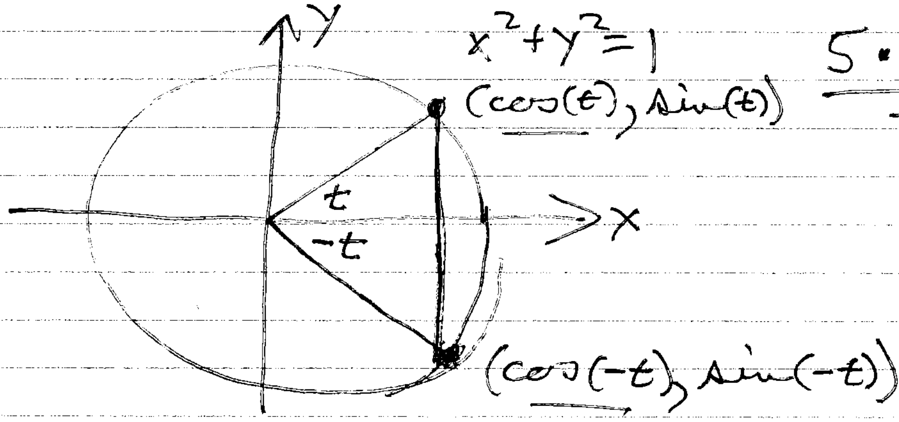
$$= \frac{-x^3-x}{x^2+\cos(x)}$$ because cosine is even

$$= -\frac{x^3+x}{x^2+\cos(x)} = -f(x) \quad \text{ODD}$$

so $$\int_{-\pi}^{\pi} \frac{x^3+x}{x^2+\cos(x)}\,dx = 0$$

$$\frac{(-1)(x^3+x)}{x^2+\cos(x)}$$

$$\frac{ab}{c} = a\frac{b}{c}$$



$$\frac{5\cdot 2}{7} = 5\cdot\frac{2}{7}$$

so $\cos(-t) = \cos(t)$

$\sin(-t) = -\sin(t)$

$$I = \int t^2 \sqrt{t+3}\, dt = \int (u-3)^2 \sqrt{u}\, du = \int (u^2 - 6u + 9) u^{1/2}\, du$$

Let $u = t+3$, $t = u-3$
$du = dt$

$$I = \int \left(u^{5/2} - 6u^{3/2} + 9u^{1/2} \right) du$$

$$= \frac{2}{7} u^{7/2} - 6 \cdot \frac{2}{5} u^{5/2} + 9 \cdot \frac{2}{3} u^{3/2} + C$$

$$= \frac{2}{7} (t+3)^{7/2} - \frac{12}{5} (t+3)^{5/2} + 6 (t+3)^{3/2} + C$$

$$\int \frac{\sec^2(\theta)}{(\tan(\theta)-1)^2}\, d\theta = \int \frac{du}{u^2} = \int u^{-2}\, du = -u^{-1} + C = -\frac{1}{(\tan(\theta)-1)} + C$$

$u = \tan(\theta) - 1$
$du = \sec^2(\theta)\, d\theta$

$$\int 5^x \sin(5^x)\, dx = \frac{1}{\ln(5)} \int \sin(u)\, du = \frac{1}{\ln(5)} \left(-\cos(u) \right) + C = -\frac{\cos(5^x)}{\ln(5)} + C$$

Let $u = 5^x$
$du = 5^x \ln(5)\, dx$
$\frac{1}{\ln(5)} du = 5^x dx$

$$\int \frac{\sin(\ln(x))}{x}\, dx = \int \sin(u)\, du = -\cos(u) + C = -\cos(\ln(x)) + C$$

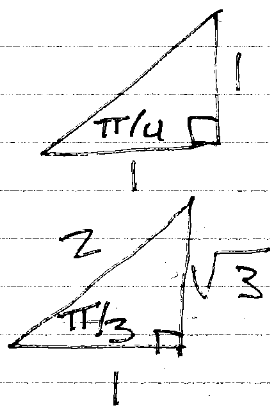
Let $u = \ln(x)$
$du = \frac{1}{x} dx$

$$\int_{1}^{\sqrt{3}} \frac{\left(\tan^{-1}(x)\right)^3}{1+x^2}\,dx = \int_{\pi/4}^{\pi/3} u^3\,du = \frac{u^4}{4}\Bigg|_{\pi/4}^{\pi/3}$$

$$u = \tan^{-1}(x)$$

$$du = \frac{1}{x^2+1}\,dx$$

x	u
1	$\pi/4$
$\sqrt{3}$	$\pi/3$



$$= \frac{1}{4}\left(\left(\frac{\pi}{3}\right)^4 - \left(\frac{\pi}{4}\right)^4\right)$$

$$= \frac{1}{4}\left(\frac{\pi^4}{81} - \frac{\pi^4}{256}\right)$$