Definite Integral

8. $\int_{-1/2}^{1/2} \sin^{-1}(x)dx$

9. $\int_{1/2}^{0} \sin^{-1}(x)dx$

10. $\int_{1}^{e} \sin(\ln(x))dx$

Combining u-substitution and IP

11. $\int x^3 \sin(x^2)dx$

Trick

12. $\int e^{x^2} x^3 dx = \int e^{a} a \cdot \frac{1}{2} da = \frac{1}{2}\left(ae^{a} - \int e^{a} da\right) = \frac{1}{2}(ae^{a} - e^{a}) + C$

$= \underline{\frac{x^2 e^{x^2} - e^{x^2}}{2} + C}$

a-sub

$a = x^2$

$da = 2x dx$

$\frac{1}{2} da = x dx$

IP: $u = a \quad dv = e^{a} da$

$du = dx \quad v = e^{a}$

Substitution: 13. $\int e^{\sqrt{x}}dx$ $= \int e^t 2t\,dt = 2\left(te^t - \int e^t dt\right) = 2(te^t - e^t) + C$

t-sub

$t = \sqrt{x}$

$dt = \frac{1}{2\sqrt{x}}dx$

$2\sqrt{x}\,dt = dx$

$2t\,dt = dx$

IP $u = t$ $dv = e^t dt$

$du = dt$ $v = e^t$

$= 2\left(\sqrt{x}e^{\sqrt{x}} - e^{\sqrt{x}}\right) + C$

Reduction: 14. Prove $\int x^n e^x dx = x^n e^x - n\int x^{n-1}e^x dx$ for integers $n \geq 1$.

IP: $u = x^n$ $dv = e^x dx$

$du = nx^{n-1}dx$ $v = e^x$

$\int x^n e^x dx = x^n e^x - \int e^x n x^{n-1} dx$

$= x^n e^x - n\int x^{n-1} e^x dx$

Use the reduction formula to find $\int x^3 e^x dx = I$

$n=3 \Rightarrow I = x^3 e^x - 3\int x^2 e^x dx$

$n=2 \Rightarrow I = x^3 e^x - 3\left(x^2 e^x - 2\int x e^x dx\right)$

$n=1 \Rightarrow I = x^3 e^x - 3x^2 e^x + 6\left(xe^x - \int e^x dx\right)$

$= x^3 e^x - 3x^2 e^x + 6xe^x - 6e^x + C$