5.5 Net change as the integral of a rate

Applications: If $F' = f$ then $\int_a^b f(x)dx = F(b) - F(a)$ so

$$\int_a^b F'(x)dx = F(b) - F(a)$$

is the total change of F from a to b.

1. Water flows into a reservoir at a rate of $2t^3 - t$ m^3/sec after t seconds. How much water flows into the reservoir from $t = 1$ to $t = 3$ seconds?

$$V = \int_1^3 (2t^3 - t)\,dt = \left[\frac{\cancel{2}t^4}{\cancel{2}4} - \frac{t^2}{2}\right]_1^3 = \frac{1}{2}\left[3^4 - 3^2 - (1^4 - 1^2)\right]$$

$$= \frac{1}{2}\left[81 - 9\right] = \frac{72}{2} = 36 \text{ m}^3$$

2. A population grows at the rate of $\frac{100}{1+t^2}$ people per month. Find the population increase, to the nearest person, from month 2 to month 10.

$$\text{pop} = \int_2^{10} \frac{100}{1+t^2}\,dt = 100 \arctan(t)\Big|_2^{10}$$

$$= 100\left(\arctan(10) - \arctan(2)\right) \approx 36 \text{ people}$$

3. An object has acceleration $a(t) = 5 - t$ m/s^2 and initial velocity of 2 m/sec. Find the total distance travelled from $t = 2$ to $t = 20$ seconds. Give your answer to one decimal place accuracy.

$$v = \int (5 - t)\,dt$$

$$= 5t - \frac{t^2}{2} + C$$

$$v(0) = 2 = C$$

$$v(t) = 5t - \frac{t^2}{2} + 2$$

Total dist travelled

$$= \int_2^{20} \left|5t - \frac{t^2}{2} + 2\right| dt$$

fnInt (abs(5x − x²/2+2), x, 2, 20)

$$\approx 488.1 \text{ m}$$

 Understand the methods so you can solve similar problems.
Understand the concepts so you can solve unfamiliar problems.

Study the (a) class notes, (b) do the 4th hour problems, (c) study the text examples, (d) do the text exercises and (e) read the next text section.

4. A particle moving in a straight line has velocity $v(t) = 2\cos(t)$ m/s.
(a) Find the displacement from times 0 s to π s.

$$\text{disp} = \int_0^{\pi} 2\cos(t)\,dt = 2\sin(t)\Big|_0^{\pi}$$

$$= 2(\sin\pi - \sin 0)$$

$$= 0$$

(b) Find the distance travelled during [0,π] s.

$$\text{dist} = \int_0^{\pi} |2\cos(t)|\,dt = 2\left[\int_0^{\pi/2} \cos(t)\,dt - \int_{\pi/2}^{\pi} \cos(t)\,dt\right]$$

Repeat using fnInt on the calculator.

$$= 2\left[\sin(t)\Big|_0^{\pi/2} - \sin(t)\Big|_{\pi/2}^{\pi}\right]$$

$$= 2[1 - 0 - (0 - 1)] = \underline{4\,m}$$

Math 9
fnInt(abs(2cos(x)), x, 0, π) ≈ 4

5. A particle accelerates at the rate $a(t) = 4 - t^2$ m/s^2 for the first 2 seconds. Estimate the distance travelled over this period if the initial velocity is 3 m/s. Give your answer to 1 decimal accuracy.

$$v = \int (4 - t^2)\,dt$$

$$= 4t - \frac{t^3}{3} + C$$

$$v(0) = 3 = C$$

$$v(t) = 4t - \frac{t^3}{3} + 3$$

$$\text{dist} = \int_0^2 \left|4t - \frac{t^3}{3} + 3\right| dt \approx \underline{12.7\,m}$$

fnInt(abs(4x − x³/3 + 3), X, 0, 2)

Omit Total versus Marginal Cost on page 325.