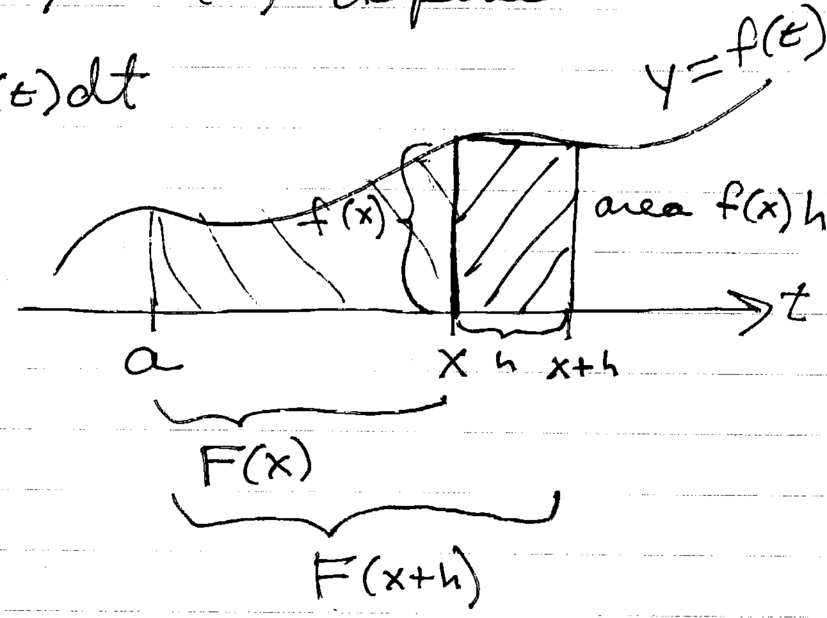
5.4 FTC part 2

Given a function $y = f(t)$ define

$$F(x) = \int_a^x f(t)\,dt$$



$$f(x)h \approx F(x+h) - F(x)$$

Divide by h: $\quad f(x) \approx \dfrac{F(x+h) - F(x)}{h}$

Take $\lim\limits_{h \to 0}$: $\quad f(x) = \lim\limits_{h \to 0}\left(\dfrac{F(x+h) - F(x)}{h}\right) = F'(x)$

<u>In summary</u> If $F(x) = \int_a^x f(t)\,dt$

then $F'(x) = f(x)$.

or $\dfrac{d}{dx}\left(\int_a^x f(t)\,dt\right) = f(x)$

Example $F(x) = \int_2^x t^3\,dt$

$$= \frac{t^4}{4}\Big|_2^x = \frac{1}{4}\left(x^4 - 2^4\right)$$

~~so~~ $F'(x) = \frac{d}{dx}\left(\frac{1}{4}(x^4 - 2^4)\right) = \frac{1}{4}\cdot 4x^3 = x^3$

Example $\frac{d}{dx}\left(\int_3^x \cos(t^3)\,dt\right) = \cos(x^3)$

Example $\frac{d}{dx}\left(\int_x^4 \sec(t^5)\,dt\right)$

$$= \frac{d}{dx}\left(-\int_4^x \sec(t^5)\,dt\right)$$

$$= -\sec(x^5)$$

P. 18 #4 $\frac{d}{dx}\int_{\cos(x)}^{3x^5} \tan(t)\,dt$

$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$$

$$= \frac{d}{dx}\left(\int_{\cos(x)}^0 \tan(t)\,dt + \int_0^{3x^5} \tan(t)\,dt\right)$$

$$= \frac{d}{dx}\left(-\int_0^{\cos(x)} \tan(t)\,dt + \int_0^{3x^5} \tan(t)\,dt\right)$$

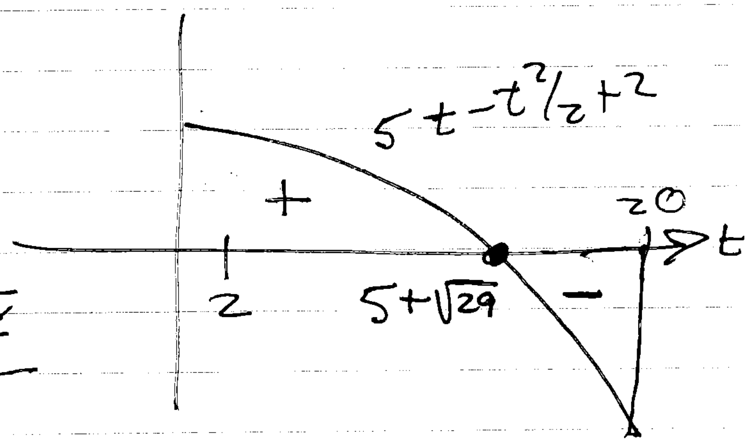
$$= -\tan(\cos(x))(-\sin(x)) + \tan(3x^5)\,15x^4$$

$$\frac{d}{dx}\left(\int_{a(x)}^{b(x)} f(t)\,dt\right) = f(b(x))\,b'(x) - f(a(x))\,a'(x)$$

fnInt   Math 9

fnInt(f(x), x, a, b)

$$I = \int_2^{20} \left|5t - \frac{t^2}{2} + 2\right| dt$$



$$-\frac{t^2}{2} + 5t + 2 = 0$$

$$t = \frac{-5 \pm \sqrt{25 - 4(-\frac{1}{2})2}}{2(-\frac{1}{2})}$$

$$= \frac{-5 \pm \sqrt{29}}{-1}$$

$$= 5 \mp \sqrt{29}$$

$$I = \int_2^{5\sqrt{29}} \left(5t - \frac{t^2}{2} + 2\right) dt - \int_{5+\sqrt{29}}^{20} \left(5t - \frac{t^2}{2} + 2\right) dt$$

5.4 The Fundamental Theorem of Calculus Part 2

$$\frac{d}{dx}\int_a^x f(t)dt = f(x)$$

$$\int_a^a f(t)dt = 0$$

1. $\frac{d}{dx}\int_0^x \frac{\sin(t)}{1+t^4}\,dt = \frac{\sin(x)}{1+x^4}$

2. Express the antiderivative $F(x)$ of $f(x) = e^{\sqrt{x}}$ satisfying $F(23) = -7$ as an integral.

$F(x) = \int_a^x e^{\sqrt{t}}\,dt$ $\quad F(x) = \int_{23}^x e^{\sqrt{t}}\,dt - 7$

$\Rightarrow F'(x) = e^{\sqrt{x}}$ $\quad F'(x) = e^{\sqrt{x}}$

The Chain Rule gives $\frac{d}{dx}\int_c^{a(x)} f(t)dt = f\big(a(x)\big)\underline{a'(x)}$.

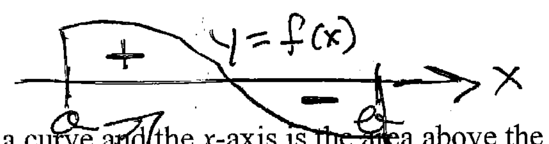
Chain rule

$$\frac{d}{dx}\big(f(g(x))\big) = f'(g(x))\,g'(x)$$

3. Given $g(x) = \int_2^{x^2} \frac{1-t}{1+t}dt$ find $g'(x)$.

$$g'(x) = \frac{1-x^2}{1+x^2}\cdot 2x$$

4. Find $\frac{d}{dx}\int_{\cos(x)}^{3x^5} \tan(t)\,dt. = \tan(3x^5)15x^4 - \tan(\cos(x))(-\sin(x))$



The **net area** between a curve and the x-axis is the area above the axis *minus* the area below and equals $\int_a^b f(x)dx$.

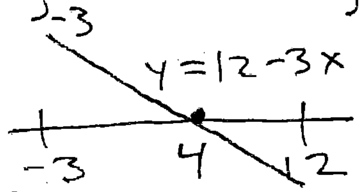
The **area** is the area above *plus* the area below, and equals $\int_a^b |f(x)|dx$.

5. For $f(x) = 12 - 3x$ find the net area between $f(x)$ and the x-axis on the interval [-3, 12].

$$\text{Net area} = \int_{-3}^{12}(12-3x)\,dx = \left[12x - \frac{3x^2}{2}\right]_{-3}^{12} = 144 - 216 - \left(-36 - \frac{27}{2}\right) = -22.5$$

6. For $f(x) = 12 - 3x$ find the area between $f(x)$ and the x-axis on the interval [-3, 12].

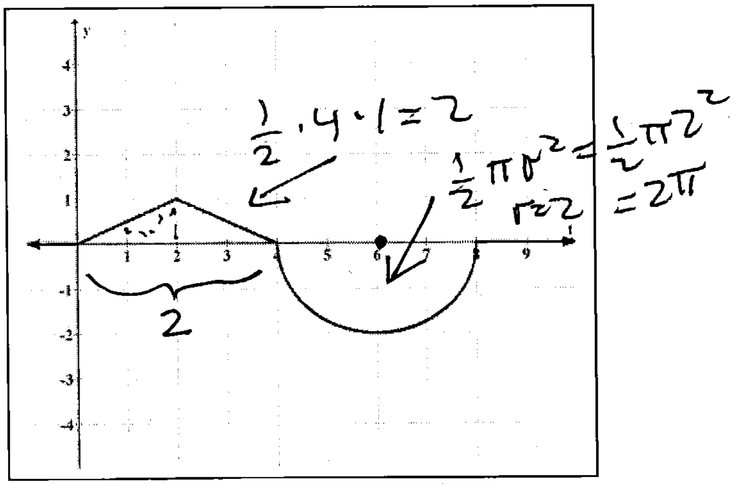
$$\text{Area} = \int_{-3}^{12}|12-3x|\,dx = \int_{-3}^{4}(12-3x)dx - \int_4^{12}(12-3x)dx = 73.5 - (-96) = 169.5$$



 Understand the methods so you can solve similar problems.
Understand the concepts so you can solve unfamiliar problems.

Study the (a) class notes, (b) do the 4th hour problems, (c) study the text examples, (d) do the text exercises and (e) read the next text section.

7. Define $G(x) = \int_0^x f(t)dt$ where $f(t)$, composed of line segments and a semicircle, is defined by the graph below.



Find $G(2) =$ 1

$\frac{1}{2} \cdot 2 \cdot 1 = 1$

$G'(2) = f(2) = 1$

$G'(6) =$ $f(6) = -2$

$G(9) = 2 - 2\pi$

and $G(0) =$ 0

The maximum value of G is 2 and the minimum is $2 - 2\pi$.

8. Again using $f(t)$ above, define $J(x) = \int_2^x |f(t)|dt$.

Find $J(0) =$ -1

$J(4) = 1$

$J'(6) = 2$

and $J(8) =$ $1 + 2\pi$

The maximum value of J is $1 + 2\pi$ and the minimum is -1.

9. Define $F(x) = \int_{-3}^{x}(t-1)(t+2)dt,\ x \geq -3.$

a) Find the intervals of increase of F.

$F'(x) = (x-1)(x+2)$
$= x^2+x-2$

$-2 \quad 1 \quad \to x$

$F'(x)\ +\ 0\ -\ 0\ +$

$F(x)$ LMax LMin

b) Find the local minimum(s) and local maximum(s) of F. F increases on $[-3,-2],[1,\infty)$

$L\max = F(-2) = \int_{-3}^{-2}(t-1)(t+2)dt$ | Lmin

$= \int_{-3}^{-2}(t^2+t-2)dt$

$= \left[\frac{t^3}{3}+\frac{t^2}{2}-2t\right]_{-3}^{-2} = \frac{11}{6}$

c) Find the intervals on which F is concave upward.

$F''(x) = 2x+1 > 0$

$2x > -1$

$x > -1/2$

F is concave up on $(-1/2, \infty)$

10. Define $r(x) = \int_{x/2-4}^{16}(t^3+e^t)dt$. [Observe $x/2 - 4 \neq \frac{x}{-2}$.] Find the following exactly:

$x/2-4$
$= \frac{x}{2}-4$

a) $r(8) = \int_{8/2-4}^{16}(t^3+e^t)dt = \int_0^{16}(t^3+e^t)dt$

$= \left[\frac{t^4}{4}+e^t\right]_0^{16} = \frac{16^4}{4}+e^{16}-(0+1)$

$= 16383+e^{16}$

b) $r'(8)$

$r'(x) = x^3+e^x,\ r'(8) = 8^3+e^8$

c) $\frac{d}{dx}r(12)$

$r(12)$ is a constant so $\frac{d}{dx}r(12) = 0$

11. Find a function f and a number $a > 0$ such that $5 - \int_a^x \frac{f(t)}{t+1}dt = x^3$.

$\frac{d}{dx}\left(5-\int_a^x \frac{f(t)}{t+1}dt\right) = \frac{d}{dx}(x^3)$

$-\frac{f(x)}{x+1} = 3x^2,\ f(x) = -3x^2(x+1)$

$5 - \int_a^x \frac{-3t^2\cancel{(t+1)}}{\cancel{t+1}}dt = x^3$

$5+\int_a^x 3t^2 = x^3$

$5+t^3\Big|_a^x = x^3$

$5+x^3-a^3 = x^3$

$a^3 = 5$

$a = \sqrt[3]{5}$

 Understand the methods so you can solve similar problems.
Understand the concepts so you can solve unfamiliar problems.

Study the (a) class notes, (b) do the 4th hour problems, (c) study the text examples, (d) do the text exercises and (e) read the next text section.