5.3 The Fundamental Theorem of Calculus

If f is continuous on $[a,b]$ and F is any antiderivative of f on $[a,b]$ then

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

<u>Notation</u> $$\int_a^b f(x)\,dx = F(x)\Big|_a^b = F(b) - F(a)$$

<u>Ex</u> $$\int_1^4 x^2\,dx = \frac{x^3}{3}\Big|_1^4 = \frac{1}{3}\left(4^3 - 1^3\right) = \frac{1}{3}(64-1)$$

$$= \frac{1}{3}(63) = 21$$

or $$\int_1^4 x^2\,dx = \left(\frac{x^3}{3}+5\right)\Big|_1^4 = \left(\frac{4^3}{3}+5\right) - \left(\frac{1^3}{3}+5\right)$$

$$= \frac{64}{3} + \cancel{5} - \frac{1}{3} - \cancel{5} = \frac{63}{3} = 21$$

$$\int_0^3 2^x a^{x+1}\,dx$$

$m^x n^x = (mn)^x$

$\int a^x\,dx = \frac{a^x}{\ln(a)} + C$ $\rightarrow a>0,\ a \neq 1$

$a^{x+1} = a^x a$

$$= \int_0^3 2^x a^x a\,dx$$

$$= a\int_0^3 2^x a^x\,dx$$

$$= a\int_0^3 (2a)^x\,dx$$

$$= \begin{cases} a\,\dfrac{(2a)^x}{\ln(2a)}\Big|_0^3 & \text{if } a \neq \frac{1}{2} \\ \frac{1}{2}\,x\Big|_0^3 & \text{if } a = \frac{1}{2} \end{cases} = \begin{cases} \dfrac{a}{\ln(2a)}\left((2a)^3 - 1\right) & \text{for } a \neq \frac{1}{2} \\ \frac{1}{2}(3-0) = \frac{3}{2} & \text{if } a = \frac{1}{2} \end{cases}$$

5.3 Evaluating the definite integral

FUNDAMENTAL THEOREM OF CALCULUS: If a function *f* is continuous on the closed interval *[a,b]* and *F* is any antiderivative of *f*, then $\int_a^b f(x)dx = F(b) - F(a)$.

1. $\int_1^2 e^{3x}dx = \left.\frac{e^{3x}}{3}\right|_1^2 = \frac{1}{3}\left(e^6 - e^3\right)$

2. Find the area under $y = 8 - x^2$ from 0 to 2.

$$\text{Area} = \int_0^2 (8-x^2)dx \qquad \frac{40}{3}$$

$$= \left(8x - \frac{x^3}{3}\right)_0^2 = \left(16 - \frac{8}{3}\right) - 0 = \frac{40}{3}$$

3. $\int_{-5}^{3} \frac{dx}{x}$ DNE, Improper integral

$$\int_1^3 \frac{dx}{x} = \ln|x|\Big|_1^3 = \ln 3 - \underbrace{\ln 1}_{=0} = \ln(3)$$

4. $\int_1^2 \frac{x^2+5x-\sqrt{x}}{x}dx = \int_1^2 (x^2+5x-\sqrt{x})\,x^{-1}dx$

$$= \int_1^2 (x + 5 - x^{-1/2})dx$$

$$= \left[\frac{x^2}{2} + 5x - 2x^{1/2}\right]_1^2 = 2 + 10 - 2\sqrt{2} - \left(\frac{1}{2} + 5 - 2\right)$$

$$= \frac{17}{2} - 2\sqrt{2}$$

5. $\int_1^4 \frac{dt}{t^{1/3}}$

$$= \int_1^4 t^{-1/3}dt = \left.\frac{3}{2}t^{2/3}\right|_1^4 = \frac{3\cdot\sqrt[3]{16}}{2} - \frac{3}{2}$$

 Understand the methods so you can solve similar problems.
Understand the concepts so you can solve unfamiliar problems.

Study the (a) class notes, (b) do the 4th hour problems, (c) study the text examples, (d) do the text exercises and (e) read the next text section.

Indefinite Integrals, or Antiderivatives

If $F'(x) = f(x)$ for all x in the domain of F then $\int f(x)dx = F(x) + C$.

$\int \sec^2(x)dx = \tan(x) + C$

$\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1}(x) + C$

$\int \sec(x)\tan(x)dx = \sec(x) + C$

$\int \frac{dt}{t^5} = \int t^{-5}dt = \frac{t^{-4}}{-4} + C = -\frac{1}{4t^4} + C$

$\int \frac{dx}{\sqrt{x}} = 2\sqrt{x} + C$

$$\int x^n dx = \begin{cases} \frac{x^{n+1}}{n+1} + C & \text{if } n \neq -1 \\ \ln|x| + C & \text{if } n = -1 \end{cases}$$

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Study the (a) class notes, (b) do the 4th hour problems, (c) study the text examples, (d) do the text exercises and (e) read the next text section.