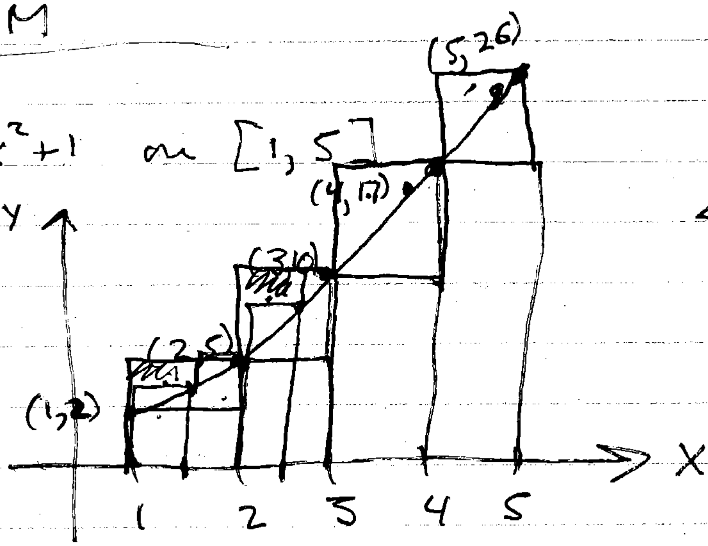
5.1 The Definite Integral

AREA
RAM

$f(x) = x^2+1$ on $[1,5]$

$$\Delta x = \frac{5-1}{n}$$



$n=4$ $\begin{cases} A_{left} = 1\cdot 2 + 1\cdot 5 + 1\cdot 10 + 1\cdot 17 = 34 \\ A_{right} = 1\cdot 5 + 1\cdot 10 + 1\cdot 17 + 1\cdot 26 = 58 \end{cases}$

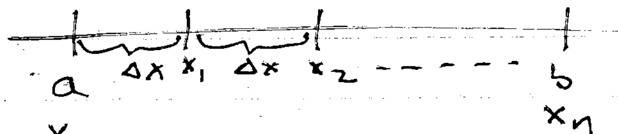
$$34 < A < 58$$

A = area under $f(x) = x^2+1$ for $1 \le x \le 5$.

Given
$y = f(x)$ on $[a,b]$, $f(x) \geq 0$ on $[a,b]$

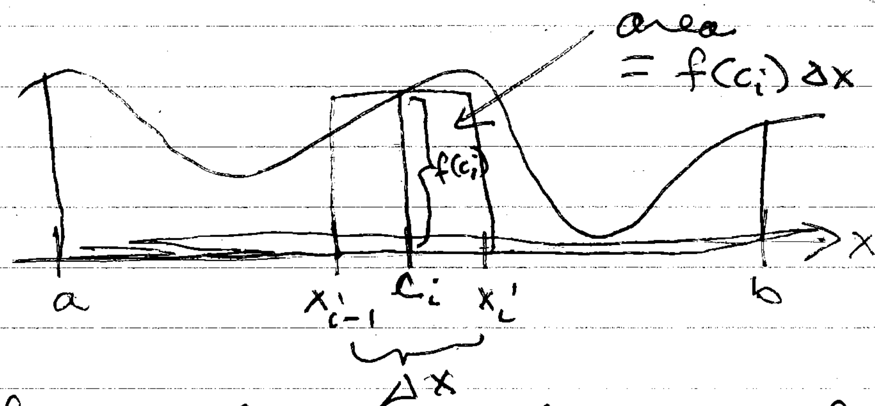
Define n a positive integer (number of rectangles)
$\Delta x = \frac{b-a}{n}$ (width of a subinterval)

Partition $[a,b]$ with



$x_0 = a$
$x_1 = a + \Delta x$
$x_2 = a + 2\Delta x$
$\vdots$
$x_i = a + i\Delta x$
$\vdots$
$x_n = a + n\Delta x = a + n\left(\frac{b-a}{n}\right) = b$

Let c_i in $[x_{i-1}, x_i]$
for $i = 1, 2, \ldots, n$
Area of a rectangle $= f(c_i)\Delta x$



$$\text{Area} \approx f(c_1)\Delta x + f(c_2)\Delta x + \cdots + f(c_i)\Delta x + \cdots + f(c_n)\Delta x$$
$$= (f(c_1) + f(c_2) + \cdots + f(c_i) + \cdots + f(c_n))\Delta x$$
$$= \sum_{i=1}^{n} f(c_i)\Delta x$$

$$\text{Area} = \lim_{n \to \infty} \left(\sum_{i=1}^{n} f(c_i)\Delta x \right)$$
$$\text{or } \lim_{n\to\infty} \left(\Delta x \sum_{i=1}^{n} f(c_i) \right) \text{ or } \lim_{n\to\infty} \left(\left(\sum_{i=1}^{n} f(c_i) \right) \Delta x \right)$$

Summation Notation

$$3(-1) + 3(0) + 3(1) + 3(2) + 3(3) = 15$$
$$= \sum_{i=-1}^{3} 3i$$

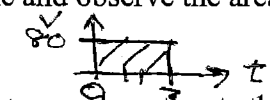
$$\sum_{j=2}^{6} 2^j = 2^2 + 2^3 + 2^4 + 2^5 + 2^6$$

$$\sum_{k=0}^{n-1} x_k = x_0 + x_1 + x_2 + \cdots + x_{n-1}$$

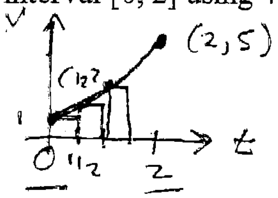
5.1 Rectangle Approximation Method

Velocity and distance travelled.

a) A car travels at a steady velocity of 80 km/hr for 3 hours. The total distance travelled is 240 km. Graph velocity against time and observe the area under the velocity line is 240.



1. If the velocity of a particle is $v = t^2+1$ m at t seconds, estimate the total distance travelled over the interval [0, 2] using 4 rectangles and (a) left endpoints;



$\Delta x = \frac{2-0}{4} = \frac{1}{2}$

$v(\frac{1}{2}) = \frac{1}{4}+1 = \frac{5}{4}$

$V(\frac{3}{2}) = \frac{9}{4}+1 = \frac{13}{4}$

$L_4 = D \simeq 1 \cdot \frac{1}{2} + \frac{5}{4} \cdot \frac{1}{2} + 2 \cdot \frac{1}{2} + \frac{13}{4} \cdot \frac{1}{2}$

$= \frac{30}{4} \cdot \frac{1}{2} = \frac{15}{4}$ m

(b) Right endpoints. 5.75 m

(c) What are the grid points? 4.625 m

AREA

Observe the area under the curve is the total distance travelled.

2. Use a calculator to estimate the area (which is equal to the distance travelled) using the AREA program or the RAM program for Riemann sums with the values of n shown below:

n	L_n	Midpoint_n	R_n
4	3.75	4.625	5.75
10	4.28	4.66	5.08
25	4.5088	4.6656	4.8288
100	4.6268	4.6666	4.7068

3. Express the area of the region under $y = 11 - x^2$ on [0, 3] as the limit of a sum. Do not evaluate the limit.

$\Delta x = \frac{b-a}{n} = \frac{3-0}{n} = 3/n$

$x_i = a + i\Delta x$

$= 0 + i \cdot 3/n = 3i/n$

$f(x_i) = 11 - \left(\frac{3i}{n}\right)^2$

$= 11 - \frac{9i^2}{n^2}$

$f(x_i)\Delta x = \left(11 - \frac{9i^2}{n^2}\right)\frac{3}{n}$

$\text{Area} = \lim_{n\to\infty} \left(\sum_{i=1}^{n} \left(11 - \frac{9i^2}{n^2}\right)\frac{3}{n}\right)$

 Understand the methods so you can solve similar problems.
Understand the concepts so you can solve unfamiliar problems.

Study the (a) class notes, (b) do the 4th hour problems, (c) study the text examples, (d) do the text exercises and (e) read the next text section.

Sigma notation

4. (a) $\sum_{i=1}^{4}(2i-1)$

16

(b) $\sum_{n=0}^{5} n^2$

55

(c) $\sum_{p=1}^{3}\frac{p+4}{5-p}$ $= \frac{5}{4}+\frac{6}{3}+\frac{7}{2}$

$= \frac{15+24+42}{12}$

$\frac{27}{4}$

$= \frac{81}{12} = \frac{\cancel{3}\cdot 27}{\cancel{3}\cdot 4} = \frac{27}{4}$

Omit Computing area as the limit of approximations on pp. 292 – 295,

 Understand the methods so you can solve similar problems.
Understand the concepts so you can solve unfamiliar problems.

Study the (a) class notes, (b) do the 4th hour problems, (c) study the text examples, (d) do the text exercises and (e) read the next text section.