

1.a) By experimenting, find a solution to the differential equation $y'' + y = 0$.

$$\begin{array}{l}
 y = \sin(x) \\
 y' = \cos(x) \\
 y'' = -\sin(x)
 \end{array}
 \quad \text{or} \quad
 \begin{array}{l}
 y = \cos(x) \\
 y' = -\sin(x) \\
 y'' = -\cos(x)
 \end{array}
 \quad \text{so } y = A \sin(x) + B \cos(x)$$

where A and B are arbitrary constants.

b) Can you, similarly, find a solution to $y'' - y = 0$?

$$\begin{array}{l}
 y = e^x \\
 y' = e^x \\
 y'' = e^x
 \end{array}
 \quad \text{or} \quad
 \begin{array}{l}
 y = e^{-x} \\
 y' = -e^{-x} \\
 y'' = e^{-x}
 \end{array}
 \quad \text{so } y = A e^x + B e^{-x}, \quad A, B \text{ arbitrary}$$

2. Show that one solution to the differential equation $y'' + 9y = 0$ is $y = \cos(3x)$.

$$\begin{array}{l}
 y' = -\sin(3x) \cdot 3 \\
 y'' = -\cos(3x) \cdot 9
 \end{array}$$

$$\text{so } y'' + 9y = -\cos(3x) \cdot 9 + 9 \cos(3x) = 0$$

3. Show that one solution to the differential equation $y'' - 9y = 0$ is $y = e^{3x}$. Can you find other solutions?

$$\begin{array}{l}
 x^2 - 9 = 0 \\
 x = \pm 3
 \end{array}$$

e^{-3x} is another solution.

The general solution is $A e^{3x} + B e^{-3x}$, A, B arbitrary