

Mathematics 126

Fourth 8.4, Taylor polynomials

1. For $f(x) = \frac{1}{2x+1}$ find the following:

a) the third-order Taylor polynomial centered at $a = 0$.

$f(x) = (2x+1)^{-1}$

$$f(0) = 1$$

$$f'(x) = -(2x+1)^{-2} \cdot 2, f'(0) = -2$$

$$f''(x) = 4(2x+1)^{-3} \cdot 2, f''(0) = 8$$

$$f'''(x) = -24(2x+1)^{-4} \cdot 2, f'''(0) = -48$$

$$P_3(x) = 1 - 2x + \frac{8x^2}{2} - \frac{48}{6}x^3$$

$$P_3(x) = 1 - 2x + 4x^2 - 8x^3$$

b) use the polynomial to estimate $\frac{1}{1.97}$.

$$1.97 = 2x + 1$$

$$2x = 0.97$$

$$x = 0.485$$

$$\frac{1}{1.97} \approx P_3(0.485) = 1 - 2(0.485) + 4(0.485)^2 - 8(0.485)^3$$

$$= 0.50761$$

(constant with $\frac{1}{1.97} \approx 0.50761$)

c) the 4th-order Taylor polynomial centered at $a = -1$.

$$f(-1) = -1$$

$$f'(-1) = -2(-1)^{-2} = -2$$

$$f''(-1) = 8(-1)^{-3} = -8$$

$$f'''(-1) = -48(-1)^{-4} = -48$$

$$f^{(4)}(x) = 96(2x+1)^{-5} \cdot 2$$

$$f^{(4)}(-1) = 96(-1)^{-5} \cdot 2 = -384$$

$$P_4(x) = -1 - 2(x+1) - \frac{8}{2}(x+1)^2 - \frac{48}{6}(x+1)^3 - \frac{384}{24}(x+1)^4$$

$$P_4(x) = -1 - 2(x+1) - 4(x+1)^2 - 8(x+1)^3 - 16(x+1)^4$$

d) use the polynomial to estimate $-\frac{1}{1.04}$.

$$\frac{1}{2x+1} = -\frac{1}{1.04}$$

$$1.04 = -2x - 1$$

$$2x = -2.04$$

$$x = -1.02$$

$$P_4(-1.02) \approx -0.96153856$$

$$\left(-\frac{1}{1.04} \approx -0.9615384615 \right)$$

2. Use a 5th-order Taylor polynomial to estimate $\sqrt[3]{124}$.

Use $f(x) = x^{1/3}$ at $a = 5^3 = 125$

$f(x) = x^{1/3}$	$f(5^3) = 5$	$\frac{f^{(n)}(5^3)}{n!}$
$f'(x) = \frac{1}{3}x^{-2/3}$	$f'(5^3) = \frac{1}{75}$	$\frac{5}{1/75}$
$f''(x) = -\frac{2}{9}x^{-5/3}$	$f''(5^3) = -\frac{2}{9 \cdot 5^5}$	$\frac{-1}{9(5^5)}$
$f^{(3)}(x) = \frac{10}{27}x^{-8/3}$	$f^{(3)}(5^3) = \frac{10}{27(5^8)}$	$\frac{2}{81(5^8)}$
$f^{(4)}(x) = -\frac{80}{81}x^{-11/3}$	$f^{(4)}(5^3) = \frac{-80}{81(5^{11})}$	$\frac{-20}{81(5^{11})246}$
$f^{(5)}(x) = \frac{880}{243}x^{-14/3}$	$f^{(5)}(5^3) = \frac{880}{243(5^{14})}$	$\frac{20}{243(5^{14})126}$

87 Understand the methods so you can solve similar problems.

Understand the concepts so you can solve unfamiliar problems.

Study the (a) class notes, (b) text examples, (c) do the text exercises, and (d) do the 4th hour problems.

$$P_5(x) = 5 + \frac{1}{75}(x-5^3) - \frac{1}{9(5^5)}(x-5^3)^2 + \frac{2}{81(5^8)}(x-5^3)^3 - \frac{20}{81(5^{11})6}(x-5^3)^4 + \frac{22}{729(5^{14})}(x-5^3)^5$$

$$P_5(124) \approx 4.98663$$