Mathematics 126 **Fourth 7.6, Improper Integrals**

Evaluate the integrals:

$$\int_{-\infty}^{3}\frac{x}{(x^2+9)^4}\,dx = \lim_{t\to-\infty}\int_t^3 \frac{x}{(x^2+9)^4}\,dx = \lim_{t\to-\infty}\frac{1}{2}\int_{t^2+9}^{18} u^{-4}\,du = \frac{1}{2}\lim_{t\to-\infty}\left(\frac{u^{-3}}{-3}\right)_{t^2+9}^{18}$$

$u = x^2+9$, $du = 2x\,dx$, $\frac{1}{2}du = x\,dx$

x	u
t	t^2+9
3	18

$$= -\frac{1}{6}\lim_{t\to-\infty}\left(\frac{1}{18^3} - \underbrace{\frac{1}{(t^2+9)^3}}_{\to 0}\right) = \frac{-1}{6(18)^3} = \frac{-1}{34992}$$

$$\int_0^1 \frac{\ln(x)}{x}\,dx = \lim_{t\to 0^+}\int_t^1 \frac{\ln(x)}{x}\,dx = \lim_{t\to0^+}\int_{\ln(t)}^{0} u\,du = \lim_{t\to0^+}\left[\frac{u^2}{2}\right]_{\ln(t)}^{0}$$

$u = \ln(x)$, $du = \frac{1}{x}dx$

x	u
t	$\ln(t)$
1	0

$$= \lim_{t\to0^+}\frac{1}{2}\left[0 - (\ln(t))^2\right] = -\infty$$

The integral diverges.

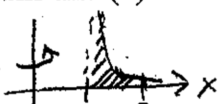
Use the Comparison Test to determine whether the integral is convergent or divergent:

$$\int_1^{\infty}\frac{x^3}{x^6+1}\,dx$$

$\frac{x^3}{x^6+1} \le \frac{x^3}{x^6} = \frac{1}{x^3}$, $\int_1^{\infty}\frac{1}{x^3}dx$ converges by the p-test.

So $\int_1^{\infty}\frac{x^3}{x^6+1}\,dx$ converges by the comparison test.

Find the volume of the solid S obtained by rotating the region R bounded by the function $f(x) = \frac{1}{(x-2)^{3/4}}$ and the x-axis on the interval (2,3] (a) about the y-axis and (b) about the x-axis. First sketch the region R.



(a) Shells, $V = 2\pi\int_2^3 x\cdot\frac{1}{(x-2)^{3/4}}\,dx = 2\pi\lim_{t\to2^+}\int_t^3\frac{x}{(x-2)^{3/4}}\,dx$

$u = x-2$, $du = dx$

x	u
t	t-2
3	1

$$V = 2\pi\lim_{t\to2^+}\int_{t-2}^{1}(u+2)u^{-3/4}\,du = 2\pi\lim_{t\to2^+}\int_{t-2}^{1}(u^{1/4}+2u^{-3/4})\,du$$

$$V = \lim_{t\to2^+}2\pi\left[\frac{4}{5}u^{5/4} + 2\cdot4u^{1/4}\right]_{t-2}^{1} = 2\pi\lim_{t\to2^+}\left[\frac{4}{5}+8-\frac{4}{5}\underbrace{(t-2)^{5/4}}_{\to0} - 8\underbrace{(t-2)^{1/4}}_{\to0}\right] = 2\pi\left(\frac{44}{5}\right) = \frac{88\pi}{5}$$

 Understand the methods so you can solve similar problems.
Understand the concepts so you can solve unfamiliar problems.
Study the (a) class notes, (b) text examples, (c) do the text exercises, and (d) do the 4[th] hour problems.

(b) Washers or Disks: $V = \pi\int_2^3\frac{1}{(x-2)^{3/2}}\,dx = \pi\lim_{t\to2^+}\int_t^3(x-2)^{-3/2}\,dx = \pi\lim_{t\to2^+}\left[-2(x-2)^{-1/2}\right]_t^3$

$$= \pi(-2)\lim_{t\to2^+}\left[1 - \frac{1}{\sqrt{t-2}}\right] = \infty.$$ The integral diverges.