

Mathematics 126

Fourth 7.5, Partial Fractions

Evaluate the integrals:

1. $\int \frac{x+4}{(x-3)(x+7)} dx = I$

$$\frac{x+4}{(x-3)(x+7)} = \frac{A}{x-3} + \frac{B}{x+7}$$

$$x+4 = A(x+7) + B(x-3)$$

$x=3: 7 = 10A, A = 7/10$

$x=-7: -3 = -10B, B = 3/10$

$$I = \frac{7}{10} \int \frac{dx}{x-3} + \frac{3}{10} \int \frac{dx}{x+7}$$

$$= \frac{7}{10} \ln|x-3| + \frac{3}{10} \ln|x+7| + C$$

2. $\int \frac{x^2-3x+1}{x^3+x^2+9x+9} dx = I$

$$x^3+x^2+9x+9$$

$$= (x^3+9x) + (x^2+9)$$

$$= x(x^2+9) + 1(x^2+9)$$

$$= (x+1)(x^2+9)$$

$$\frac{x^2-3x+1}{x^3+x^2+9x+9} = \frac{A}{x+1} + \frac{Bx+C}{x^2+9}$$

$$x^2-3x+1 = A(x^2+9) + (x+1)(Bx+C)$$

$x=-1: 5 = 10A, A = 1/2$

$x=0: 1 = \frac{1}{2}(9) + 1C, C = -7/2$

$x=1: -1 = \frac{1}{2}(10) + 2(B-7/2)$

$$-6 = 2B - 7$$

$$1 = 2B$$

$$B = 1/2$$

$$I = \frac{1}{2} \int \frac{dx}{x+1} + \frac{1}{2} \int \frac{x}{x^2+9} dx - \frac{7}{2} \int \frac{dx}{x^2+9}$$

$$= \frac{1}{2} \ln|x+1| + \left\{ \begin{array}{l} u = x^2+9 \\ du = 2x dx \\ \frac{1}{2} du = x dx \end{array} \right. - \frac{7}{2} \cdot \frac{1}{3} \tan^{-1}\left(\frac{x}{3}\right) + C$$

so $\frac{1}{2} \int \frac{x}{x^2+9} dx = \frac{1}{4} \int \frac{du}{u}$

$$= \frac{1}{4} \ln|u| + C$$

$$= \frac{1}{4} \ln(x^2+9) + C$$

so $I = \frac{1}{2} \ln|x+1|$

$$+ \frac{1}{4} \ln(x^2+9)$$

$$- \frac{7}{6} \tan^{-1}\left(\frac{x}{3}\right) + C$$