Mathematics 126, Fourth 7.1

Evaluate the integrals:

1. $\int t^3 \ln(t)\, dt$ $= \frac{1}{4}t^4 \ln(t) - \frac{1}{4}\int t^4 \cdot \frac{1}{t}\, dt = \frac{1}{4}t^4\ln(t) - \frac{1}{4}\int t^3 dt$

$\underline{IP}$ $dv = t^3 dt$, $u = \ln(t)$

$v = \frac{t^4}{4}$ $\quad du = \frac{1}{t}dt$

$= \frac{1}{4}t^4 \ln(t) - \frac{1}{16}t^4 + C$

2. $\int e^{-2x}\cos(x)\, dx$ $= I = e^{-2x}\sin(x) + 2\int e^{-2x}\sin(x)\,dx$

$\underline{IP}$ $u = e^{-2x}$ $\quad dv = \cos(x)dx$ $\quad\big|\quad u = e^{-2x}$ $\quad dv = \sin(x)dx$

$du = -2e^{-2x}dx$ $\quad v = \sin(x)$ $\quad\big|\quad du = -2e^{-2x}dx$ $\quad v = -\cos(x)$

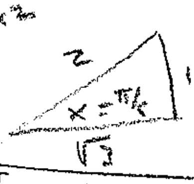
$I = e^{-2x}\sin(x) + 2\left(-e^{-2x}\cos(x) - 2I\right)$

$5I = e^{-2x}\sin(x) - 2e^{-2x}\cos(x)$, $I = \frac{e^{-2x}}{5}\left(\sin(x) - 2\cos(x)\right) + C$

3. $\int_0^{1/2} \sin^{-1}(x)dx$ $= \left(x\sin^{-1}(x)\right)\Big|_0^{1/2} - \int_0^{1/2}\frac{x}{\sqrt{1-x^2}}dx = \frac{1}{2}\cdot\frac{\pi}{6} - 0 - \left(-\int_1^{3/4}\frac{1}{2}\frac{du}{\sqrt{u}}\right)$

$\underline{IP}$ $u = \sin^{-1}(x)$ $\quad dv = dx$

$du = \frac{1\, dx}{\sqrt{1-x^2}}$ $\quad v = x$



$\underline{Sub}$ $u = 1-x^2$, $du = -2x\,dx$, $-\frac{1}{2}du = x\,dx$

x	u
0	1
$\frac{1}{2}$	$\frac{3}{4}$

$= \frac{\pi}{12} + \frac{1}{\cancel{2}}\cdot\cancel{2}\,u^{1/2}\Big|_1^{3/4}$

$= \frac{\pi}{12} + \left(\frac{\sqrt{3}}{2} - 1\right)$

$= \frac{\pi}{12} + \frac{\sqrt{3}}{2} - 1$

4. $\int x5^x dx$ $= I$

$\underline{IP}$ $u = x$ $\quad dv = 5^x dx$

$du = dx$ $\quad v = \frac{5^x}{\ln(5)}$

$I = \frac{x5^x}{\ln(5)} - \int\frac{5^x}{\ln(5)}dx$

$= \frac{x5^x}{\ln(5)} - \frac{5^x}{(\ln(5))^2} + C$

 Understand the methods so you can solve similar problems.
Understand the concepts so you can solve unfamiliar problems.

Study the (a) class notes, (b) do the 4th hour problems, (c) study the text examples, (d) do the text exercises and (e) read the next text section.