

Mathematics 126, Fourth 6.5, Work

1. A spring has a natural length of 0.9 m. A force of 15 N stretches the spring to a length of 1.4 m. Find the work required to stretch the spring 2 m beyond its natural length.

$$F(0.5) = .5k = 15$$

$$k = 30, F(x) = 30x$$

$$W = \int_0^2 30x \, dx = 15x^2 \Big|_0^2 = \underline{60 \text{ J}}$$

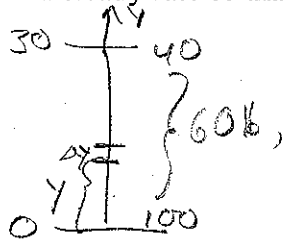
2. An 8 lb bucket is lifted 30 ft up by pulling on a rope that weighs 0.07 lb/ft. (a) How much work is required?

Bucket: $P(30) = 240 \text{ ft}\cdot\text{lb}$

Rope: $\Delta w = (30-y)(0.07) \Delta y$
 $w = \int_0^{30} (30-y)(0.07) \, dy = 345 \text{ ft}\cdot\text{lb}$

$$\begin{array}{r} 240 \\ 31.5 \\ \hline 271.5 \text{ ft}\cdot\text{lb} \end{array}$$

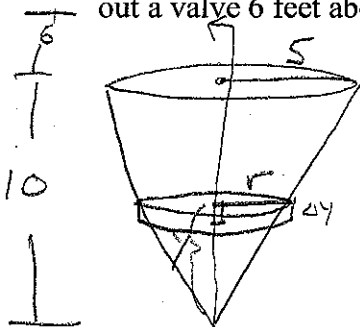
(b) What if the bucket is filled with a fluid weighing 100 lb at the bottom, which leaks at a steady rate so that the fluid weighs 40 lb at the top?



leaks 2 lb/ft
 At height y , fluid weighs $100 - 2y \text{ lb}$
 To be moved a distance Δy
 $\Delta w = (100 - 2y) \Delta y$
 $w = \int_0^{30} (100 - 2y) \, dy = 2100$

$$\begin{array}{r} 271.5 \\ 2100 \\ \hline 2371.5 \text{ ft}\cdot\text{lb} \end{array}$$

3. A tank is in the shape of an inverted circular cone with height 10 ft and radius 5 ft. The tank is filled with oil weighing 52 lb/ft³. How much work is done to pump the oil out a valve 6 feet above the top of the tank?



$$V = \pi r^2 \Delta y, \quad \frac{r}{y} = \frac{5}{10}, \quad r = \frac{1}{2}y$$

$$V = \pi \left(\frac{1}{2}y\right)^2 \Delta y$$

$$= \frac{\pi}{4} y^2 \Delta y$$

$$D = 16 - y$$

$$W = \int_0^{10} \frac{\pi}{4} y^2 (52) (16 - y) \, dy \approx \underline{115,715.329 \text{ ft}\cdot\text{lb}}$$