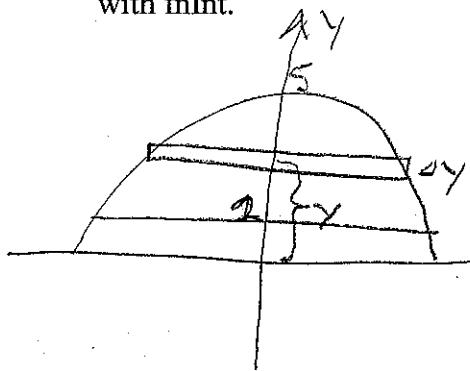


Mathematics 126,

Fourth 6.4, Cylindrical Shells

1. Set up an integral expression using shells for the exact volume of a solid remaining after drilling a hole of radius 2 through the centre of a sphere of radius 5. Approximate with fnInt.



$$x^2 + y^2 = 25$$

$$x = \pm \sqrt{25 - y^2}$$

$$h(y) = 2\sqrt{25 - y^2}$$

$$p(y) = y$$

$$V = 2\pi \int_2^5 y \cdot 2\sqrt{25 - y^2} dy$$

$$\approx \underline{\underline{128.312\pi}}$$

2. Set up an integral expression using shells for the exact volume of the solid obtained by rotating the region above the x axis and bounded by $y = x$, $y = -x$, $y = 5 - x^2$ about the line shown below.

(a) $y = 25$. Approximate with fnInt.

$$5 - x^2 = x$$

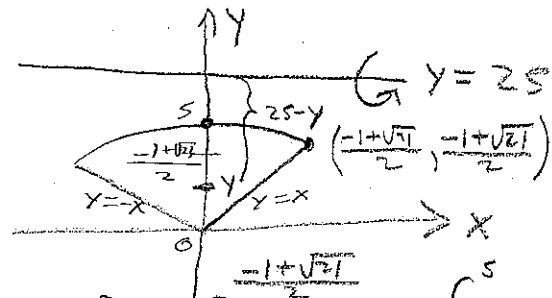
$$0 = x^2 + x - 5$$

$$x = \frac{-1 \pm \sqrt{1 - 4(-5)}}{2}$$

$$= \frac{-1 \pm \sqrt{21}}{2}$$

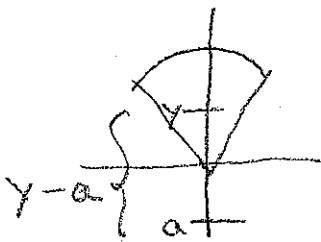
$$x^2 = 5 - y$$

$$x = \pm \sqrt{5 - y}$$



By symmetry, $V = 2 \left[2\pi \int_0^5 y(25-y) dy + \int_{\frac{-1+\sqrt{21}}{2}}^5 \sqrt{5-y}(25-y) dy \right]$

(b) $y = a$, $-4 < a < 0$.



$$V = 2 \left[2\pi \int_0^5 y(x-a) dy + \int_{\frac{-1+\sqrt{21}}{2}}^5 \sqrt{5-y}(y-a) dy \right]$$