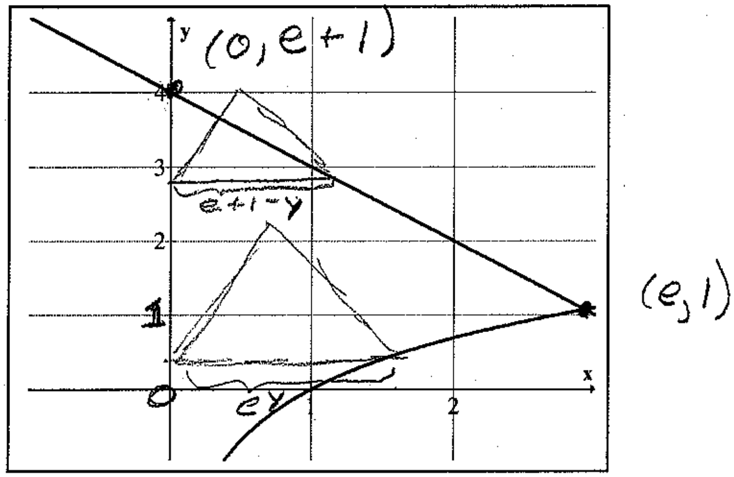
Mathematics 126 Fourth 6.3, Volume

Consider the region **R** bounded by the x-axis, y-axis, y = ln(x), y = -x + e + 1. Set up an integral expression (but do not evaluate) without absolute values to give the exact volume of the solid:

1.a) with base **R** and whose cross sectional areas parallel to the x-axis are equilateral triangles;



① $y = \ln(x),\ x = e^{y}$

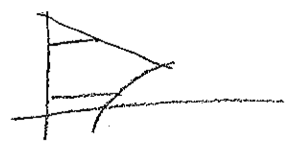
$A_{\Delta} = \dfrac{b^2\sqrt{3}}{4} = \dfrac{e^{2y}\sqrt{3}}{4}$

② $A_{\Delta} = \dfrac{(e+1-y)^2\sqrt{3}}{4}$

③ $V = \displaystyle\int_0^1 \left(\frac{e^{2y}\sqrt{3}}{4}\right)dy + \int_1^{e+1} \frac{(e+1-y)^2\sqrt{3}}{4}\,dy$

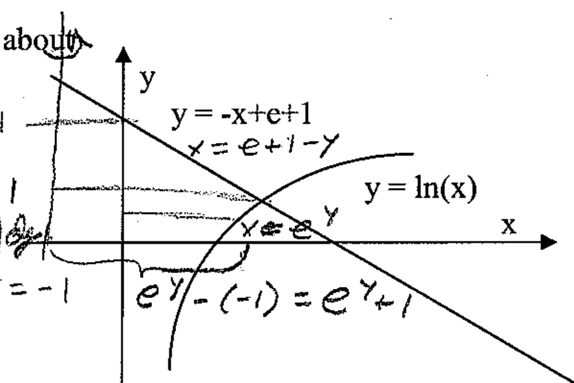
(b) with base **R** and whose cross sectional areas parallel to the x-axis are squares;

$V = \displaystyle\int_0^1 e^{2y}\,dy + \int_1^{e+1} (e+1-y)^2\,dy$



2. Volume of the solid obtained by rotating **R** about

(a) x = -1



$0 \le y \le 1,\ R(y) = e^{y}+1,\ r(y) = 1$

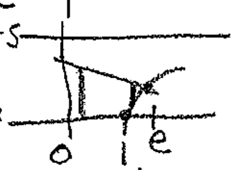
$1 \le y \le e+1,\ R(y) = e+2-y,\ r(y) = 1$

$V = \pi\displaystyle\int_0^1 \left[(e^{y}+1)^2 - 1^2\right]dy + \pi\int_1^{e+1}\left[(e+2-y)^2 - 1^2\right]dy$

(b) y = 5

$0 \le x \le 1,\ R(x) = 5,\ r(y) = 5-(-x+e-1) = 4+x-e$

$1 \le x \le e,\ R(x) = 5-\ln(x),\ r(x) = 4+x-e$

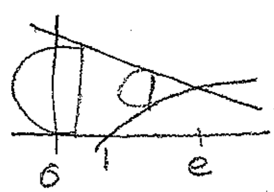


$V = \pi\displaystyle\int_0^1 \left[5^2 - (4+x-e)^2\right]dx + \pi\int_1^{e}\left[(5-\ln(x))^2 - (4+x-e)^2\right]dx$

3. Volume of the solid whose base is **R** and whose cross-sections perpendicular to the x-axis are semicircles. $A = \frac{1}{2}\pi r^2$

$0 \le x \le 1,\ r = \dfrac{e+1-x}{2},\ A = \dfrac{\pi}{8}(e+1-x)^2$

$1 \le x \le e,\ r = \dfrac{e+1-x-\ln(x)}{2},\ A = \dfrac{\pi}{8}(e+1-x-\ln(x))^2$



 Understand the methods so you can solve similar problems.
Understand the concepts so you can solve unfamiliar problems.
Study the (a) class notes, (b) text examples, (c) do the text exercises, and (d) do the 4th hour problems.

$V = \displaystyle\int_0^1 \frac{\pi}{8}(e+1-x)^2\,dx + \int_1^{e} \frac{\pi}{8}(e+1-x-\ln(x))^2\,dx$