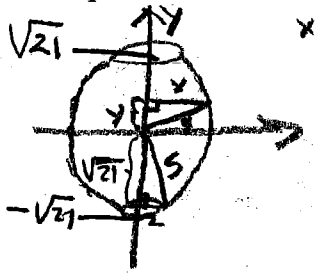
Mathematics 126 **Fourth 6.2**

1. Find the volume of a solid remaining after boring a cylindrical hole of radius 2 m out of a sphere of radius 5 m.



$x^2+y^2=25$

$x=\sqrt{25-y^2}$

$A=\pi r^2=\pi x^2$

$=\pi(25-y^2)$

$A_2=\pi 2^2=4\pi$

$V=\int_{-\sqrt{21}}^{\sqrt{21}}(\pi(25-y^2)-4\pi)\,dy$

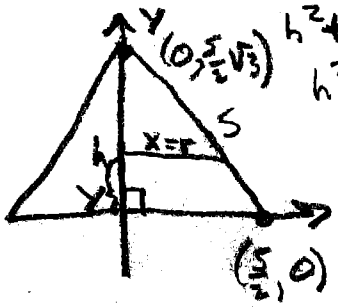
$=2\pi\int_0^{\sqrt{21}}(21-y^2)\,dy$

$=2\pi\left(21y-\frac{y^3}{3}\right)_0^{\sqrt{21}}=2\pi\left(21^{3/2}-\frac{21^{3/2}}{3}\right)$

$=2\pi\left(\frac{2\cdot 21^{3/2}}{3}\right)$

$=\frac{4\pi}{3}\cdot 21^{3/2}$

2. Find the volume of a solid whose base is an equilateral triangle of side 5 m and whose cross sections parallel to one side of the triangle are semicircles.



$h^2+\left(\frac{5}{2}\right)^2=5^2$

$h^2=25-\frac{25}{4}=\frac{3\cdot 25}{4}$

$h=\frac{5}{2}\sqrt{3}$

$m=\frac{\frac{5}{2}\sqrt{3}-0}{0-5/2}=-\sqrt{3}$

$y=-\sqrt{3}x+\frac{5}{2}\sqrt{3}$

$x=\frac{y-\frac{5}{2}\sqrt{3}}{-\sqrt{3}}=\frac{5}{2}-\frac{y}{\sqrt{3}}$

$A(y)=\frac{1}{2}\pi r^2=\frac{1}{2}\pi\left(\frac{5}{2}-\frac{y}{\sqrt{3}}\right)^2$

$V(y)=\int_0^{\frac{5}{2}\sqrt{3}}\frac{\pi}{2}\left(\frac{5}{2}-\frac{y}{\sqrt{3}}\right)^2 dy \approx 8.502$ ft/in

3. Find the average value of $f(x)=\sqrt[3]{x}$ *on* $[0,10]$.

$$\bar{f}=\frac{1}{10-0}\int_0^{10}x^{1/3}\,dx=\frac{1}{10}\cdot\frac{3}{4}x^{4/3}\Big|_0^{10}=\frac{3}{4}\cdot\frac{10^{4/3}}{10}=\frac{3\sqrt[3]{10}}{4}$$

4. Find the point(s) at which the value of $f(x)=\frac{e^x}{r}, r>0$ equals the average value of the function on the interval $[0,r]$.

① $\bar{f}=\frac{1}{r-0}\int_0^r\frac{e^x}{r}\,dx$

$=\frac{1}{r^2}e^x\Big|_0^r$

$=\frac{1}{r^2}(e^r-1)$

② $f(a)=\frac{e^a}{r}$

③ $\frac{e^a}{r}=\frac{1}{r^2}(e^r-1)$

$e^a=\frac{1}{r}(e^r-1)$

$a=\ln\left(\frac{e^r-1}{r}\right)$

$f(a)=\frac{e^{\ln\left(\frac{e^r-1}{r}\right)}}{r}=\frac{e^r-1}{r^2}$

④ Point is $\left(\ln\left(\frac{e^r-1}{r}\right),\frac{e^r-1}{r^2}\right)$

 Understand the methods so you can solve similar problems.
Understand the concepts so you can solve unfamiliar problems.

Study the (a) class notes, (b) do the 4th hour problems, (c) study the text examples, (d) do the text exercises and (e) read the next text section.