Mathematics 126 **Fourth 5.7**

1. $\int \frac{x+7}{\sqrt{36-x^2}}dx = \int \frac{x}{\sqrt{36-x^2}}dx + 7\int \frac{dx}{\sqrt{36-x^2}} = -\frac{1}{2}\int \frac{du}{\sqrt{u}} + \frac{7}{6}\int \frac{dx}{\sqrt{1-\left(\frac{x}{6}\right)^2}}$

Sub, $u = 36-x^2$, $du = -2x\,dx$, $-\frac{1}{2}du = x\,dx$

Sub, $v = \frac{x}{6}$, $dv = \frac{1}{6}dx$, $6\,dv = dx$

$= -\frac{1}{2}\cdot 2u^{1/2} + \frac{7}{6}\int \frac{6\,dv}{\sqrt{1-v^2}}$

$= -\sqrt{36-x^2} + 7\sin^{-1}(v) + C$

$= -\sqrt{36-x^2} + 7\sin^{-1}\left(\frac{x}{6}\right) + C$

2. $\int 3^x e^{8x}dx$

$= \int e^{x\ln 3}e^{8x}dx$

$= \int e^{(8+\ln 3)x}dx = \frac{e^{(8+\ln 3)x}}{8+\ln 3} + C$

3. $\int \frac{dx}{5+7x^2}$

$= \frac{1}{5}\int \frac{dx}{1+\left(x\sqrt{\frac{7}{5}}\right)^2}$, $u = x\sqrt{\frac{7}{5}}$, $du = dx\sqrt{\frac{7}{5}}$, $\sqrt{\frac{5}{7}}du = dx$

$\frac{1}{5}\int \frac{\sqrt{\frac{5}{7}}du}{1+u^2} = \frac{1}{\sqrt{35}}\tan^{-1}\left(x\sqrt{\frac{7}{5}}\right) + C$

4. $\int \frac{\sin^{-1}(e^x)}{\sqrt{1-e^{2x}}}e^x dx = \int u\,du = \frac{u^2}{2} + C = \frac{1}{2}\left(\sin^{-1}(e^x)\right)^2 + C$

$u = \sin^{-1}(e^x)$

$du = \frac{1}{\sqrt{1-e^{2x}}}e^x dx$

5. $\int (3x^2+1)\sqrt{1-2x}\,dx = \int \left(3\left(\frac{1-u}{2}\right)^2+1\right)\sqrt{u}\left(-\frac{1}{2}du\right) = -\frac{1}{2}\int\left(\frac{3}{4}(1-2u+u^2)+1\right)u^{\frac{1}{2}}du$

$u = 1-2x$, $du = -2dx$, $-\frac{1}{2}du = dx$

$2x = 1-u$

$x = \frac{1-u}{2}$

$= -\frac{1}{2}\int\left(\frac{7}{4}u^{\frac{1}{2}} - \frac{3}{2}u^{3/2} + \frac{3}{4}u^{5/2}\right)du = -\frac{1}{2}\left(\frac{7}{4}\cdot\frac{2}{3}u^{3/2} - \frac{3}{2}\cdot\frac{2}{5}u^{5/2} + \frac{3}{4}\cdot\frac{2}{7}u^{7/2}\right) + C$

$= -\frac{1}{4}(1-2x)^{3/2} + \frac{3}{10}(1-2x)^{5/2} - \frac{3}{28}(1-2x)^{7/2} + C$

6. Suppose $b > 1$. Find all numbers $a < 1$, in terms of b, such that $\int_a^b \frac{\ln(x)}{x}dx = 0$.

$\int_a^b \frac{\ln(x)}{x}dx = \int_{\ln(a)}^{\ln(b)} u\,du = \left.\frac{u^2}{2}\right|_{\ln(a)}^{\ln(b)} = \frac{1}{2}\left((\ln(b))^2 - (\ln(a))^2\right) = 0$

$u = \ln(x)$, $du = \frac{1}{x}dx$

x	u
a	$\ln(a)$
b	$\ln(b)$

$(\ln(b))^2 = (\ln(a))^2$

$\ln(b) = \pm\ln(a)$

and $\ln(b) = \ln(a) \Rightarrow a = b$, extraneous

$\ln(b) = -\ln(a) = \ln(a^{-1}) \Rightarrow \underline{a = b^{-1}}$

 Understand the methods so you can solve similar problems.
Understand the concepts so you can solve unfamiliar problems.

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Study the (a) class notes, (b) text examples, (c) do the text exercises, (d) do the 4th hour problems and (e) read the next text section.