Math 126 Fourth 5.6 Substitution

Evaluate the integrals:

1. $\int \frac{ln(x^2)}{x}\,dx = 2\int \frac{\ln(x)}{x}\,dx = 2\int u\,du = u^2 + C = (\ln(x))^2 + C$

   $u = \ln(x),\ du = \frac{1}{x}\,dx$

2. $\int \frac{(\ln(x))^2}{x}\,dx = \int u^2\,du = u^3/3 + C = \frac{(\ln(x))^3}{3} + C$

   $u = \ln(x),\ du = \frac{1}{x}\,dx$

3. $\int tan^4(x)sec^2(x)\,dx = \int u^4\,du = u^5/5 + C = \frac{\tan^5(x)}{5} + C$

   $u = \tan(x),\ du = \sec^2(x)\,dx$

4. $\int_0^3 x\sqrt{4-x}\,dx = \int_4^1 (4-u)\sqrt{u}\,(-du) = -\int_4^1 (4u^{1/2} - u^{3/2})\,du = I$

   $u = 4-x,\ du = -dx,\ -du = dx$

   $x = 4-u$

x	u
0	4
3	1

   $I = -\left(4\cdot\frac{2}{3}u^{3/2} - \frac{2}{5}u^{5/2}\right)\Big|_4^1$

   $= -\left[\frac{8}{3} - \frac{2}{5} - \left(\frac{8}{3}\cdot 8 - \frac{2}{5}\cdot 32\right)\right]$

   $= \underline{94/15}$

5. $\int_{-8}^8 x^5 e^{x^2}\,dx$

   $f(x) = x^5 e^{x^2}$

   $f(-x) = (-x)^5 e^{(-x)^2} = -x^5 e^{x^2} = -f(x)$

   so f is odd, $\int_{-8}^8 f(x)\,dx = \underline{0}$

6. $\int_0^{\sqrt{2}} x\sqrt{4-x^4}\,dx = \frac{1}{2}\underbrace{\int_0^2 \sqrt{4-u^2}\,du}_{1/4\text{ circle, } r=2} = \frac{1}{2}\cdot\frac{1}{4}\cdot\pi 2^2 = \frac{\pi}{2}$

   $u = x^2$

   $du = 2x\,dx$

x	u
0	0
$\sqrt{2}$	2

7. $\int_0^1 \frac{sin^{-1}(x)}{\sqrt{1-x^2}}\,dx = \int_0^{\pi/2} u\,du = \frac{u^2}{2}\Big|_0^{\pi/2} = \frac{1}{2}\left(\frac{\pi}{2}\right)^2 = \frac{\pi^2}{8}$

   $u = \sin^{-1}(x)$

   $du = \frac{1}{\sqrt{1-x^2}}\,dx$

x	u
0	0
1	$\pi/2$