Mathematics 126 Fourth 5.3

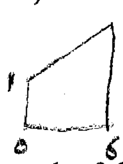
1. Use geometric properties to evaluate the following integrals:

a) $\int_{-2}^{2}\left(4-\sqrt{4-x^2}\right)dx = \int_{-2}^{2} 4\,dx - \int_{-2}^{2}\sqrt{4-x^2}\,dx$ ← semicircle, $r=2$, $A = \frac{1}{2}\pi r^2 = \frac{1}{2}\pi 2^2 = 2\pi$

$= 4(4) - 2\pi$

$= 16 - 2\pi$

b) $\int_{0}^{6}(2x+1)dx = \frac{1+13}{2}\cdot 6 = 7\cdot 6 = 42$

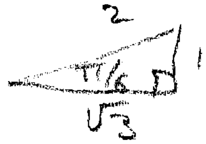


Trapezoid, $A = \frac{a+b}{2}h$

2. Evaluate the following integrals using the Fundamental Theorem of Calculus:

a) $\int_{0}^{\pi}\left(5x^2+\cos(x)\right)dx = \left[\frac{5x^3}{3} + \sin(x)\right]_0^{\pi} = \frac{5\pi^3}{3} + \sin(\pi) - (0 + \sin(0)) = \frac{5\pi^3}{3}$

b) $\int_{0}^{0.5}\left(\frac{2}{\sqrt{1-x^2}} + e^{-3x}\right)dx = \left[2\sin^{-1}(x) - \frac{e^{-3x}}{-3}\right]_0^{0.5} = 2\sin^{-1}(0.5) + \frac{e^{-3/2}}{3} - \left(2\sin^{-1}(0) + \frac{e^0}{3}\right)$



$= 2\cdot\frac{\pi}{6} + \frac{e^{-3/2}}{3} - \frac{1}{3}$

$= \frac{\pi + e^{-3/2} - 1}{3}$

3. Find the indefinite integrals:

a) $\int(2+t^3)(2t-5)dt$

$= \int(2t^4 - 5t^3 + 4t - 10)dt$

$= 2\frac{t^5}{5} - 5\frac{t^4}{4} + 2t^2 - 10t + C$

b) $\int\left(x\sqrt[3]{x} + \sec^3(x)\underbrace{\frac{\tan(x)}{1+\tan^2(x)}}_{\sec^2(x)}\right)dx$

$= \int\left(x^{4/3} + \sec(x)\tan(x)\right)dx = \frac{3}{7}x^{7/3} + \sec(x) + C$

 Understand the methods so you can solve similar problems.
Understand the concepts so you can solve unfamiliar problems.
Study the (a) class notes, (b) text examples, (c) do the text exercises, and (d) do the 4th hour problems.