

Mathematics 126 Fourth 5.1

1. Estimate the area under $f(x) = \frac{1}{2x}$ from $x = 1$ to $x = 3$ using four approximating

rectangles, no calculator, and (a) right endpoints; $\Delta x = \frac{b-a}{n} = \frac{3-1}{4} = \frac{1}{2}$
 $A \approx [f(\frac{3}{2}) + f(2) + f(\frac{5}{2}) + f(3)] \frac{1}{2} = 0.475$

(b) left endpoints;

$$A \approx (\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5}) \frac{1}{2} = \frac{77}{120}$$

(c) midpoints.

$$A \approx [f(\frac{5}{4}) + f(\frac{7}{4}) + f(\frac{9}{4}) + f(\frac{11}{4})] \frac{1}{2} = \frac{1888}{3465}$$

2. Use a calculator to estimate the distance travelled from $t = 1$ to $t = 5$ by a vehicle with velocity $v(t) = \ln(t^2 + 1)$ metres/second. Give answers accurate to 3 decimal places.

$$y_i = \ln(x^2 + 1) \\ \text{RAM, } A=1, B=5$$

n	10	100	500
left endpoint	8.252	8.722	8.763
right endpoint	9.278	8.825	8.784

Observe that as n increases, the values using left and right endpoints converge.

$$3. \sum_{i=0}^5 \frac{i^2-3}{2i+1} = \frac{-3}{1} + \frac{-2}{3} + \frac{1}{5} + \frac{6}{7} + \frac{13}{9} + \frac{22}{11} = \frac{263}{315}$$

4. Express the limit as a definite integral.

(a) $\lim_{n \rightarrow \infty} \left(\sum_{i=1}^n \frac{\cos(1+5i/n)}{2+5i/n} \cdot \frac{5}{n} \right)$, $\Delta x = \frac{5}{n}$, $b-a=5$. If $a=0$, $b=5$. $f(x) = \frac{\cos(1+x)}{2+x}$
 $\int_0^5 \frac{\cos(1+x)}{2+x} dx$

(b) $\lim_{n \rightarrow \infty} \left(\sum_{i=1}^n \ln(5 + \frac{\pi-3}{n}i) \cdot \frac{\pi-3}{n} \right)$, $\Delta x = \frac{\pi-3}{n} = \frac{b-a}{n}$. If $a=3$, $b=\pi$.
 $f(x) = \ln(2+x)$, $\int_3^\pi \ln(2+x) dx$

5. Express the definite integral as a limit, in accordance with the definition of the definite integral.

$$\int_{-1}^2 \frac{\ln(x^2+12)}{1+x^4} dx = \lim_{n \rightarrow \infty} \left(\sum_{i=1}^n \frac{\ln \left[\left(-1 + \frac{3}{n}i\right)^2 + 12 \right]}{1 + \left(-1 + \frac{3}{n}i\right)^4} \right) \left(\frac{3}{n} \right)$$

6. For $f(x) = x^2 - 3x$ on $[1, 5]$ and find a Riemann sum with n intervals using

(a) left endpoints; $\Delta x = \frac{5-1}{n} = 4/n$, $a_i = 1 + \frac{4i}{n}$, $\sum_{i=0}^{n-1} \left[\left(1 + \frac{4i}{n}\right)^2 - 3\left(1 + \frac{4i}{n}\right) \right] \frac{4}{n}$

(b) right endpoints.

$$\sum_{i=1}^n \left[\left(1 + \frac{4i}{n}\right)^2 - 3\left(1 + \frac{4i}{n}\right) \right] \frac{4}{n}$$

11 Understand the methods so you can solve similar problems.

Understand the concepts so you can solve unfamiliar problems.



Study the (a) class notes, (b) text examples, (c) do the text exercises, (d) do the 4th hour problems and (e) read the next text section.