Mathematics 126 **Fourth 10.6, Power series**

Find the radius of convergence and open interval of convergence of the series

1. $\sum_{n=1}^{\infty}\frac{(-3)^n(5x-2)^n}{(n+1)^2}$. Ratio Test $\lim_{n\to\infty}\left|\frac{3^{n+1}(5x-2)^{n+1}}{(n+2)^2}\frac{(n+1)^2}{3^n(5x-2)^n}\right|$

$= \lim_{n\to\infty}\left|3(5x-2)\left(\frac{n+1}{n+2}\right)^2\right| = 3|5x-2| < 1$

$|5x-2| < 1/3$

$-1/3 < 5x-2 < 1/3$

$5/3 < 5x < 7/3$

$1/3 < x < 7/15$

$R = \frac{\frac{7}{15}-\frac{1}{3}}{2} = 1/15$

$I = \left(\frac{1}{3}, 7/15\right)$

2. $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{(2n+1)!}(x-2)^n$. Ratio Test $\lim_{n\to\infty}\left|\frac{(x-2)^{n+1}}{(2n+3)!}\frac{(2n+1)!}{(x-2)^n}\right| = \lim_{n\to\infty}\left|\frac{x-2}{(2n+3)(2n+2)}\right|$

$= 0,\ R = \infty,\ I = (-\infty,\infty)$

3. $\sum_{n=1}^{\infty}\left(\frac{(-3x-a)^n}{5^{2n}}\right)$. Ratio Test $\lim_{n\to\infty}\left|\frac{(3x+a)^{n+1}}{5^{2(n+1)}}\frac{5^{2n}}{(3x+a)^n}\right| = \lim_{n\to\infty}\left|\frac{3x+a}{5^2}\right| < 1$

$|3x+a| < 25,\ -25 < 3x+a < 25$

$-25-a < 3x < 25-a$

$\frac{-25-a}{3} < x < \frac{25-a}{3}$

$R = \frac{\frac{25-a}{3}-\left(\frac{-25-a}{3}\right)}{2} = \frac{25}{3}$

$I = \left(\frac{-25-a}{3}, \frac{25-a}{3}\right)$

4. $\sum_{n=1}^{\infty}\frac{(x+6)^n n}{2n+1}$ Ratio test $\lim_{n\to\infty}\left|\frac{(x+6)^{n+1}(n+1)}{2n+3}\frac{2n+1}{(x+6)^n n}\right|$

$= \lim_{n\to\infty}\left|(x+6)\frac{2n+1}{2n+3}\cdot\frac{n+1}{n}\right| = |x+6| < 1$

$-1 < x+6 < 1$

$-7 < x < -5$

$R = 1,\ I = (-7,-5)$

5. $\sum\frac{\ln(3+e^n)}{5n}x^n$ Ratio test $\lim_{n\to\infty}\left|\frac{\ln(3+e^{n+1})x^{n+1}}{5(n+1)}\cdot\frac{5n}{\ln(3+e^n)x^n}\right| = \lim_{n\to\infty}\left|\frac{\ln(3+e^{n+1})}{\ln(3+e^n)}\frac{n}{n+1}x\right| = |x| < 1$

$\frac{n+1}{n} \to 1$

$R = 1,\ I = (-1,1)$

6. Find a power series representation and the radius of convergence for $f(x) = \ln(1-2x)$

$\boxed{\frac{1}{1-x} = \sum x^n,\ |x|<1}$

$f'(x) = \frac{1(-2)}{1-2x}$

$= -2\sum_{0}^{\infty}(2x)^n,\ |2x| < 1 = \sum_{0}^{\infty} -2^{n+1}x^n,\ |x| < \frac{1}{2}$

$f(x) = \int f'(x)dx = \sum_{0}^{\infty} -2^{n+1}\frac{x^{n+1}}{n+1} + C$

$f(0) = \ln(1) = 0 = C$ ~~so~~

$f(x) = \sum_{0}^{\infty}\frac{(-1)2^{n+1}}{n+1}x^{n+1} + C$

$f(x) = \frac{x}{2x+3}\ \frac{1/3}{1/3}$

$= \frac{x}{3}\ \frac{1}{\frac{2}{3}x+1} = \frac{x}{3}\ \frac{1}{1-(-\frac{2}{3}x)} = \frac{x}{3}\sum_{0}^{\infty}\left(-\frac{2}{3}x\right)^n$

$= \sum_{0}^{\infty}\frac{(-1)^n 2^n x^{n+1}}{3^{n+1}}$ for $\left|-\frac{2}{3}x\right| < 1,\ |x| < \frac{3}{2}$

 ~~Understand the methods so~~ you can solve similar problems.
Understand the concepts so you can solve unfamiliar problems.

Study the (a) class notes, (b) text examples, (c) do the text exercises, (d) do the 4th hour problems and (e) read the next text section.