

10.5 The Ratio Test

Omit the Root Test.

Ratio Test: If $\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| < 1$ then $\sum a_n$ converges.

If $\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| > 1$ then $\sum a_n$ diverges.

If $\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = 1$ then the test is inconclusive so use another test.

1. Determine if the following series converge or diverge.

(a) $\sum_1^{\infty} \frac{12^n}{(n+1)5^n}$ Ratio test $\lim_{n \rightarrow \infty} \left| \frac{12^{n+1}}{(n+2)5^{n+1}} \cdot \frac{(n+1)5^n}{12^n} \right| = \lim_{n \rightarrow \infty} \left| \frac{12}{5} \cdot \frac{(n+1)}{(n+2)} \right| = \frac{12}{5} > 1$
 The series diverges

(b) $\sum n^4 e^{-n}$ Ratio test $\lim_{n \rightarrow \infty} \left| \frac{(n+1)^4 e^n}{e^{n+1} n^4} \right| = \lim_{n \rightarrow \infty} \left| \left(\frac{n+1}{n} \right)^4 \cdot \frac{1}{e} \right|$
 $= \frac{1}{e} < 1$, The series converges.

(c) $\sum_{n=1}^{\infty} \frac{n}{(n+1)!}$ Ratio test $\lim_{n \rightarrow \infty} \left| \frac{n+1}{(n+2)!} \cdot \frac{(n+1)!}{n} \right| = \lim_{n \rightarrow \infty} \left| \frac{n+1}{n} \cdot \frac{1}{n+2} \right|$
 $= 0 < 1$. The series converges.

(d) $\sum_{n=1}^{\infty} \frac{\ln(n)}{n!}$ Ratio test $\lim_{n \rightarrow \infty} \left| \frac{\ln(n+1)}{(n+1)!} \cdot \frac{n!}{\ln(n)} \right| = \lim_{n \rightarrow \infty} \left| \frac{\ln(n+1)}{\ln(n)} \cdot \frac{1}{n+1} \right|$
 $= 0 < 1$. The series converges.

2. Find all values of τ for which the following series converges: $\sum_{k=1}^{\infty} \frac{\tau^k k}{(k+1)^k}$

Ratio test $\lim_{k \rightarrow \infty} \left| \frac{\tau^{k+1} (k+1)}{(k+2)^{k+1}} \cdot \frac{(k+1)^k}{\tau^k k} \right| = \lim_{k \rightarrow \infty} \left| \tau \left(\frac{k+1}{k} \right) \left(\frac{k+1}{k+2} \right)^k \frac{1}{k+2} \right| = 0$
 The series converges for all τ .

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 Understand the concepts so you can solve unfamiliar problems.

Study the (a) class notes, (b) text examples, (c) do the text exercises, (d) do the 4th hour problems and (e) read the next text section.