Mathematics 126 Fourth 10.4, Alternating series test

Determine whether the series converges or diverges, stating the test you use. If the series converges, find the exact sum if possible; otherwise find the smallest value of n such that the sum of the first n terms is in error by less than 0.001.

(a) $\sum_{n=1}^{\infty}(-1)^n \frac{3^n}{(2n+1)!}$

AST

① $\frac{3^{n+1}}{(2n+3)!} < \frac{3^n}{(2n+1)!}$

$\iff 3 < (2n+3)(2n+2)$ ✓

② $\lim_{n\to\infty} \frac{3^n}{(2n+1)!} = \lim_{n\to\infty}\left(\underbrace{\frac{3\cdot 3}{1\cdot 3}}_{\le 3}\cdot\underbrace{\frac{3}{5}\cdots\frac{3}{(2n+1)}}_{\text{each factor}<1}\right) \le \lim_{n\to\infty} 3\left(\frac{3}{2n+1}\right) = 0$ ✓

So the series converges.

Take $u(n) = 3^n/(2n+1)!$

and $u(4) \simeq 2.2E-4$

so $n = 3$

(b) $\sum(-1)^n \frac{3n-1}{2n+1}$

Divergence test

$\lim_{n\to\infty} |a_n| = \lim_{n\to\infty} \frac{3n-1}{2n+1}\cdot\frac{1/n}{1/n} = \lim_{n\to\infty}\frac{3-\overset{\to 0}{1/n}}{2+\underset{\to 0}{1/n}} = 3/2 \neq 0$

The series diverges.

(c) $\sum \frac{(-10)^n}{n!}$

AST

① $\frac{10^{n+1}}{(n+1)!} > \frac{10^n}{n!}$

$\iff 10 > n+1$ ✓

So true for all $n = 10, 11, 12, \ldots$

② $\lim_{n\to\infty}\frac{10^n}{n!} = \lim_{n\to\infty}\left(\underbrace{\frac{10\cdots 10}{1\cdots 10}}_{=C}\cdot\underbrace{\frac{10\cdots 10}{11\cdots n}}_{\text{all factors}<1}\right) \le \lim_{n\to\infty}\left(C\cdot\frac{10}{n}\right) = 0$

So the series converges.

Take $U(n) = 10^n/n!$, $u(32) = 3.4E-4$ so $n+1 = 32$, $n = 31$.

Understand the methods so you can solve similar problems.
Understand the concepts so you can solve unfamiliar problems.

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Study the (a) class notes, (b) do the 4th hour problems, (c) study the text examples, (d) do the text exercises and (e) read the next text section.