Mathematics 126 Fourth 10.2 , Series

1. Determine if each of the following series is convergent or divergent. State the names of any tests you use. If the series converges, find the sum. Assume the sum is from n = 1 to ∞ if no limits are shown.

(a) $\sum_{1}^{\infty}\frac{3}{n} = 3\sum_{1}^{\infty}$ (b) $\sum\frac{\pi^{n+2}}{4^{n-1}}$ (c) $\sum\left(\cos\left(\frac{1}{n}\right)-\cos\left(\frac{1}{n+1}\right)\right)$

(a) Harmonic, diverges. (b) $\frac{a_{n+1}}{a_n}=\frac{\pi}{4}$, geometric, converges, sum $= \frac{\pi^3/1}{1-\pi/4} = \frac{4\pi^3}{4-\pi}$

(d) $\sum\frac{1-2n}{3n+4}$ (c) Telescoping, $\lim_{n\to\infty} S_n = \cos(1)-\cos(0)=\cos(1)-1$

(d) $\lim_{n\to\infty} a_n = \frac{-2}{3} \neq 0$. Series diverges by the divergence test.

2. The nth partial sum of the series $\sum a_n$ is $s_n = \frac{2n+1}{3n-1}$.

Find (a) a_n and (b) the sum of the series.

(a) $a_n = S_n - S_{n-1} = \frac{2n+1}{3n-1} - \frac{2n-1}{3n-3}$ (b) $S = \lim_{n\to\infty} S_n = \frac{2}{3}$

3. Determine whether the series converges or diverges. State the names of any tests you use. If the series converges find the sum exactly, or if the exact sum is not attainable, estimate the sum to 3 decimal accuracy.

(a) $\sum_{n=1}^{\infty}(\sqrt[3]{n+2}-\sqrt[3]{n+1})$ Telescoping series.

$S_n = (\sqrt[3]{3}-\sqrt[3]{2}) + (\sqrt[3]{4}-\sqrt[3]{3}) + (\sqrt[3]{5}-\sqrt[3]{4}) + \cdots + (\sqrt[3]{n+1}-\sqrt[3]{n}) + (\sqrt[3]{n+2}-\sqrt[3]{n+1})$

$S_n = \sqrt[3]{n+2}-\sqrt[3]{2}$

$S = \lim_{n\to\infty} S_n = \lim_{n\to\infty}(\sqrt[3]{n+2}-\sqrt[3]{2}) = \infty$

so the series diverges.

(b) $\sum_{k=3}^{\infty}\left(\frac{-5}{7}\right)^{2k}$, $\left(\frac{-5}{7}\right)^{2k} = \left(\frac{5}{7}\right)^{2k}$

$\frac{a_{k+1}}{a_k} = \frac{(\frac{5}{7})^{2k+2}}{(\frac{5}{7})^{2k}} = \frac{25}{49} = r$, geometric, converges

Sum $= \frac{(5/7)^6}{1-25/49} = \frac{49}{24}\left(\frac{5}{7}\right)^6 = \frac{5^6}{24(7^4)} = \frac{15625}{57624}$

(c) $\sum_{k=2}^{\infty}(0.97+\sin(k\pi))^k$ ($\sin(k\pi)$ "0")

$\frac{a_{k+1}}{a_k} = 0.97$, geometric, converges.

Sum $= \frac{.97^2}{1-.97} = \frac{94.09}{3} = 31.36\overline{3}$

(d) $\sum_{r=1}^{\infty}\left(\frac{3^r}{r^3}\right)$ $r = 1\ \ 2\ \ 3\ \ 4\ \ 5$

Not geometric $\frac{3^r}{r^3} = 3\ \ \frac{9}{8}\ \ \frac{27}{27}\ \ \frac{81}{64}\ \ \frac{243}{125}$, $f(x) = \frac{3^x}{x^3}$, $f'(x) = \frac{3^x\ln(3)x^3 - 3^x3x^2}{x^6}$

$= \frac{3^x x^2(x\ln(3)-3)}{x^6} > 0$ for $x > 3$

so series (d) diverges

4. Use your calculator to find to 5 decimal accuracy the value of the 20^{th}

(a) term of the sequence {sin(n)/n}

nMin = 1
u(n) = sin(n)/n
u(nMin) = sin(1)

$u(20) \approx 0.04565$

(b) partial sum of the series $\sum\frac{2^n}{n!}$.

Change u(n) = 2^n/n! + u(n-1)", u(nMin) = 2

$\sum_{n=1}^{20}\frac{2^n}{n!} \approx 6.38906$

 Understand the methods so you can solve similar problems.
Understand the concepts so you can solve unfamiliar problems.
Study the (a) class notes, (b) text examples, (c) do the text exercises, and (d) do the 4th hour problems.