Mathematics 126 Fourth 10.1, Sequences

1. Determine the convergence or divergence of the following sequences. If a sequence converges, find the limit: (a) $a_n = \dfrac{5n+2}{1+\sqrt{n}}$. $\lim_{n\to\infty} a_n = \lim_{x\to\infty} \frac{5x+2}{1+x^{1/2}} \overset{L'H}{=} \lim_{x\to\infty} \frac{5}{\frac{1}{2\sqrt{x}}}$

$= \lim_{x\to\infty} 10\sqrt{x} = \infty$

DIVERGES

(b) $\{\sin(n)\}$ Diverges because it oscillates between -1 and 1.

(c) $a_n = \dfrac{n(\sqrt{n}-1)}{2+5n^{3/2}}$. $\lim_{n\to\infty} a_n = \lim_{x\to\infty} \left(\frac{x^{3/2}-x}{2+5x^{3/2}}\right) = \lim_{x\to\infty} \left(\frac{1-\frac{1}{\sqrt{x}}}{\frac{2}{x^{3/2}}+5}\right) = \frac{1}{5}$

$\frac{1}{\sqrt{x}} \to 0$, $\frac{2}{x^{3/2}} \to 0$

(d) $\left\{\dfrac{2^n}{(2n+1)!}\right\}$. $a_n = \left(\frac{2}{1\cdot 3}\right)\left(\frac{2}{5}\right)\left(\frac{2}{7}\right)\cdots\left(\frac{2}{(2n+1)}\right) \to 0$. $\lim_{n\to\infty} a_n = 0$

$\frac{2}{1\cdot 3}$: $\frac{2}{3} < 1$, $\frac{2}{5} < 1$, $\frac{2}{7} < 1$, $\frac{2}{(2n+1)} < 1$

2. Find a formula for the general term a_n of the sequence

$\left\{-3, \frac{9}{2}, \frac{-27}{6}, \frac{81}{24}, \frac{-243}{120}, \ldots\right\}$

$n = 1, 2, 3, 4, 5,$ $a_n = \dfrac{(-1)^n \cdot 3^n}{n!}$

$n! = 1, 2, 6, 24, 120$

3. Determine which of the following sequences are geometric. Find the limit, if it exists, of each.

a) $a_n = 1/4^n$ $\quad \frac{a_{n+1}}{a_n} = \frac{1/4^{n+1}}{1/4^n} = \frac{4^n}{4^{n+1}} = \frac{1}{4} = r$, geometric

$\lim_{n\to\infty} \frac{1}{4^n} = 0$

b) $a_n = n + \tan(n\pi/2)$

$\frac{a_{n+1}}{a_n} = \frac{n+1+\tan((n+1)\pi/2)}{n+\tan(n\pi/2)}$ Not geometric

Since $\tan(\frac{\pi}{2})$, $\tan(\frac{3\pi}{2})$, etc. DNE, the limit does not exist.

4. For the sequence {20, 15, 10, 5, ...} find (a) an explicit formula for the sequence and (b) determine if the sequence converges and its limit, or diverges.

$\{20, 15, 10, 5, \ldots\}$

$n = 1, 2, 3, 4$

$a_n = 25 - 5n$

$\lim_{n\to\infty} a_n = -\infty$. Sequence diverges.

 Understand the methods so you can solve similar problems.
Understand the concepts so you can solve unfamiliar problems.

Study the (a) class notes, (b) text examples, (c) do the text exercises, (d) do the 4th hour problems and (e) read the next text section.