Integral Calculus
Mathematics 126

http://66.51.172.120/fharris/index.htm

Mathematics 126 - 01 in BR 205
CRN 10206

Spring 2015

Week	**Monday 1 pm**	**Thursday 10 am**
Jan 5 - 8	Area 5.1	Definite Integral 5.2
Jan 12 - 15	Fundamental Theorem 5.3	Fundamental Thm 5.4 & Net Ch 5.5
Jan 19 - 22	Substitution Method 5.6	Transcendental Functions 5.7
Jan 26 - 29	**Test 1 to 5.7;** Area 6.1	Area 6.1, Vol, Avg val 6.2
Feb 2 - 5	Volume by Disks 6.3	Volume by Cylindrical Shells 6.4
Feb 9 - 13	*No classes*	*No classes*
Feb 16 -19	Work 6.5	**Test 2 on Ch 6;** Integr by Parts 7.1
Feb 23 - 26	Trigonometric Integrals 7.2	Partial Fractions 7.5
Mar 2 - 5	Improper Integrals 7.6	Arc Length 8.1
Mar 9 - 12	Taylor Polynomials 8.4	DE 9.1, Orthogonal Traj; Mixture
Mar 16 - 19	**Test 3 on Ch 7-9;** Seq 10.1	Series 10.2
Mar 23 - 26	Alt Series 10.4 and Ratio 10.5	Power Series 10.6
Mar 30 - A 2	Taylor Series 10.7	**Optional T4 on Ch 10;**Review
Apr 6 - 8	*No classes*	Final Exam information
Apr 13 - 24	**Final Examinations, Your hard work pays off!**	

We will have 3 scheduled tests. Schedule changes will be announced in class.
There are **no makeup tests** if you miss a test.
If you miss a test, write the optional test in the last class.
Tests will start promptly at the beginning of class, so be on time.

Tests will usually last 50 minutes, and include both no-calculator and calculator portions.

 Understand the methods so you can solve similar problems.
Understand the concepts so you can solve unfamiliar problems.
Study the (a) class notes, (b) do the 4th hour problems, (c) study the text examples, (d) do the text exercises and (e) read the next text section.

Background Expectations

Mathematics 126

In the Jon Rogawski Calculus text you need to know the following:

From the plates inside the cover

1. Algebra, everything but the formula for $(x + y)^n$.

2. Trigonometry, everything but the law of sines, law of cosines, addition and subtraction formulas.

3. The elementary functions except for the hyperbolic and inverse hyperbolic functions.

4. Differentiation except for the hyperbolic and inverse hyperbolic functions.

After a section is covered in class, **study**:

1. The class notes and
2. The Fourth hour assignment with answers checked on the website and
3. The text section with close attention to the examples and
4. The assigned exercises, with answers checked in the back of the text and
5. Read the next section which we will cover in class.

Study with a friend and you will usually learn more.

Assignments Rogawski 126
STUDY GUIDE for Mathematics 126

Study Mathematics 126 for more than 1 hour every day.

Tests will be based on

- examples and review questions done in class,
- examples in the textbook (so read each section carefully paying close attention to the examples),
- exercises in the handouts and 4th hour assignments and
- exercises listed below.

After each class, study the notes from class

Second, do the 4th hour questions, *and check the answers at http://66.51.172.120/fharris*

Third, read the text carefully, write out solutions to the examples and understand the concepts.

Fourth, do the exercises listed below *and check the answers in the back of the text. If you have errors redo the question. If you need assistance, ask your instructor, or your friends, or get assistance in the Math Learning Centre, BR 289.* ***Be sure you can do each question correctly.***

Fifth, study the section of the text we will cover in the next class.

Help

Get help from Frank in office hours, or from the Math Learning Centre. Bring along the work you have done when you come for help. If you had trouble on a test or exam, bring along the work you have done and see Frank in office hours to discuss how your study time might be used more productively.

Math 126: Calculus II by Jon Rogawski 2012. Questions below are from the Exercises, not from the Preliminary Questions. The Exercises follow after the Preliminary Questions.

Section	Exercises
Chapter 5, Integration	
5.1	Omit pp 292-295. Do 1, 5, 9, 11, 15, 17, 19, 23, 25, 27, 29, 65, 67, 69, 75, and 77.
5.2	Odd numbers 1, 3, 7, 9, 13, 15, 17, 19, 23, 27, 29, 33, 37, 41, 43, 45, 51, 57, 61, 63, 65, 69, 73.

5.3 1, 7, 11, 15 ,17, 21, 23, 27, 31, 35, 39, 43, 47, 51, 53, 55, 59.
5.4 1, 3, 9, 15, 17, 19, 23, 25, 29, 33, 35, 37, 39, 41, 45, 47.
5.5 Omit 'Total versus Marginal Cost' on p. 325. Do 1, 5, 7, 11, 15, 19, 21, 23.
5.6 1, 9, 15, 23, 31, 39, 45, 49, 55, 61, 67, 71, 73, 81, 85, 89, 91.
5.7 We omit arcsecant, so omit Example 2, and Do odd numbers 1, 3, 5, 9, 11, 17, 23, 27, 33, 39, 43, 49, 55, 57, 69. [Note the answer to #7 in the SSM is incorrect.Try $tan^{-1}(\tan(8))$ on your calculator.]
Review exercises Do 1, 3, 5, 9, 11, 15, 19, 25, 27, 31, 33, 37, 41, 45, 47, 49, 55, 57, 59, 63, 67, 69, 73,75, 87, 89, 91.

Chapter 6, Applications of the Integral
6.1 1, 5, 9, 13, 17, 19, 21, 25, 29, 31, 35, 39, 43, 47, 49, 57.
6.2 1, 5 7, 9, 13, 15, 17, 19, 21, 23; omit density; Do 39, 45, 53, 55, 57.
6.3 1, 5, 11, 15, 23, 25, 27, 29, 31, 39, 45, 49, 51, 55, 57, 59.
6.4 1, 7, 11, 17, 23, 37, 41, 47, 49.
6.5 7, 11, 15, 19, 21, 23, 25, 27, 33.
Review exercises 1, 5, 9, 15, 19, 23, 25, 27, 31, 33, 35, 45, 47, 51.

Chapter 7, Techniques of Integration
7.1 1, 3, 5, 11, 15, 21, 25, 31, 37, 43, 47, 53, 67, 71, 77.
7.2 1,3,5,7, 9,13,21,23,25,27,29,31,33,35,37,41,43, 45, 47, 49, 51, 53, 55
7.5 Omit example 6 on p. 432 which involves trigonometric substitution. Do 1, 5, 9, 13, 21, 29, 37, 45, 47, 49, 65.
7.6 Omit the p-integral and comparison test from the bottom of p.440 to p.443. Do 1, 5, 9, 13, 23, 29, 33, 35, 37, 41, 45, 49, 51, 53.
Review exercises 9, 11, 13, 17, 21, 35, 39 (consider cases $a > 0$, $a = 0$, $a < 0$), 41, 43, 45, 51, 57, 63, 75, 77, 79, 83, 91.

Chapter 8, further applications of the integral and Taylor polynomials
8.1 Omit example 2 and surface area following example 3 to the end of the section. Do 1, 3, 7, 9, 15, 21 (note 15 and 21 require all the steps to evaluate improper integrals).
8.4 Omit the Error Bound from the bottom of p. 492 to the top of p. 495. Do error estimates with the GC. Do 1, 7, 13, 21, 23, 27, 49, and 53.
Review exercises 1, 3, 21, 23, 25, 27, 29 and 33.

Chapter 9, Introduction to differential equations
9.1 Study examples 1 and 2. Omit all material after example 2 to the end of the section. Do 1, 3, 5, 7, 9, 15, 21, 25, 29, 33, 37, and 43.
9.4 Do 1, 5, and 9.
Orthogonal Trajectories Do all 4 exercises.
Mixing Problems Do all 4 exercises. See my website for complete solutions.
Review exercises 1, 3, 5, 9, 23, 25, 29 and 47.
Do 46 [Answer to (c) is $\boldsymbol{t = -\frac{5}{2}\ln(0.01)}$ minutes].

Chapter 10, Infinite series

10.1 Omit Bounded Sequences pp 543 to 545. Do 1, 5, 9, 11, 13, 17, 21, 25, 27, 29, 35, 39, 45, 49, 51, 55, 59, 63, 65, 67, 69, and 75.

10.2 Do 1, 3, 7, 9, 11, 13, 15, 17, 21, 25, 29, 33, 35, 37, and 43. Although question 9 says CAS, you can do it with a TI84+.

10.4 Omit Absolute and Conditional Convergence. When the text asks if a series converges absolutely or converges conditionally we need only show the series converges or diverges. Start at Theorem 2, the Leibniz or Alternating Series Test, on p. 570. In the following exercises, simply determine if the series converges: Do 1, 3, 5, 7, 9, 11, 13, 15, 17, 25, 29, 33.

10.5 Omit the Root Test. Do 1, 5, 9, 13, 15, 21, 23, 27, 33, 43, 45, 51, 55.

10.6 We only find **open** intervals of convergence, so omit the endpoint checks (eg. In example 1 omit Step 2). Omit 'Power Series Solutions of Differential Equations' from the bottom of p. 585 to p. 588. Do the exercises (but omit the endpoint parts), 1, 3, 5, 7, 9, 11, 15, 17, 19, 23, 27, 31, 33, 45, 47, 51.

10.7 Omit example 7 and omit 'Binomial Series' from the middle of p. 596 top. 599. We are only concerned with **open** intervals on which the series converge. Do the odd numbered questions from 1, 3, 7, 9, 13, 15, 29, 31, 35, 39, 41, 45, 47, 49, 53 and 57.

Chapter Review Exercises Do 1, 3, 7, 9, 13, 17, 21, 25, 29, 33, 35, 63, 67, 69.

Capilano University Mathematics Style Guide

The following guide describes the format for giving an answer in any mathematics assignment, test or final exam. Failure to follow these guidelines may result in a penalty at the discretion of your instructor.

Graphical answers should include the following information:

Window dimensions [Xmin, Xmax] by {Ymin, Ymax]

A graph label, ie. "$Y_1 = f(x)$" to identify what graph is shown.

Where axes are visible, they must be labelled and arrows must be included to show the positive direction.

Your final answer must appear clearly identified.

The symbols "=" and "≈" must be used appropriately and not confused.
The use of an arrow sign "⇒" in place of an equal sign "=" is inappropriate.
Incorrect steps in the midst of a solution may be penalized, even if the correct answer is achieved.
Units must be given in application problems.
Variables introduced by a student to help solve a problem must be defined in either a diagram or a sentence.

Mathematics 126
Calculus 2

Chapter 5

The Integral

$$\int f(x)dx$$

 Understand the methods so you can solve similar problems.
Understand the concepts so you can solve unfamiliar problems.
Study the (a) class notes, (b) do the 4th hour problems, (c) study the text examples, (d) do the text exercises and (e) read the next text section.

5.1 Rectangle Approximation Method

Velocity and distance travelled.

a) A car travels at a steady velocity of 80 km/hr for 3 hours. The total distance travelled is ________________. Graph velocity against time and observe the area under the velocity line is ___________________.

1. If the velocity of a particle is $v = t^2+1$ m at t seconds, estimate the total distance travelled over the interval [0, 2] using 4 rectangles and (a) left endpoints;

(b) Right endpoints.

(c) What are the grid points?

Observe the area under the curve is the total distance travelled.

2. Use a calculator to estimate the area (which is equal to the distance travelled) using the AREA program or the RAM program for Riemann sums with the values of n shown below:

n	L_n	Midpoint_n	R_n
4			
10			
25			
100			

3. Express the area of the region under $y = 11- x^2$ on [0, 3] as the limit of a sum. Do not evaluate the limit.

Sigma notation

4. (a) $\sum_{i=1}^{4}(2i-1)$

(b) $\sum_{n=0}^{5} n^2$

(c) $\sum_{p=1}^{3} \frac{p+4}{5-p}$

Omit <u>Computing area as the limit of approximations</u> on pp. 292 – 295,

Mathematics 126 Fourth 5.1

1. Estimate the area under $f(x) = \dfrac{1}{2x}$ from $x = 1$ to $x = 3$ using four approximating rectangles, no calculator, and (a) right endpoints;

(b) left endpoints;

(c) midpoints.

2. Use a calculator to estimate the distance travelled from $t = 1$ to$t = 5$by a vehicle with velocity$v(t) = ln(t^2+1)$ metres/second.Give answers accurate to 3 decimal places.

n	10	100	500
left endpoint			
right endpoint			

Observe that as n increases, the values using left and right endpoints converge.

3. $\sum_{i=0}^{5} \frac{i^2-3}{2i+1}$

4. Express the limit as a definite integral.

(a) $$\lim_{n\to\infty}\left(\sum_{i=1}^{n} \frac{\cos(1+5i/n)}{2+5i/n}\frac{5}{n}\right)$$

(b) $$\lim_{n\to\infty}\left(\sum_{i=1}^{n} \ln(5+\frac{\pi-3}{n}i)\frac{\pi-3}{n}\right)$$

5. Express the definite integral as a limit, in accordance with the definition of the definite integral.

$$\int_{-1}^{2} \frac{\ln(x^2+12)}{1+x^4}\,dx$$

6. For $f(x) = x^2 - 3x$ on [1,5] and find a Riemann sum with n intervals using
(a) left endpoints;

(b) right endpoints.

 Understand the methods so you can solve similar problems.
Understand the concepts so you can solve unfamiliar problems.

Study the (a) class notes, (b) do the 4th hour problems, (c) study the text examples, (d) do the text exercises and (e) read the next text section.

5.2 Definite Integral and Area

1. Express the limit as a definite integral:

$$\lim_{n\to\infty}\left[\sum_{i=1}^{n} sin\left(2+\frac{7i}{n}\right)\frac{7}{n}\right]$$

2. Express each of the integrals as the limit of a Riemann sum:

$$\int_{3}^{17}\sqrt[3]{x^5+8}\,dx$$

$$\int_{-4}^{4}\sqrt{16-x^2}\,dx$$

$$\int_{1}^{8}(5-x)\,dx$$

 Understand the methods so you can solve similar problems.
Understand the concepts so you can solve unfamiliar problems.

Study the (a) class notes, (b) do the 4th hour problems, (c) study the text examples, (d) do the text exercises and (e) read the next text section.

3. Given $f(x) = 4 - x^2$, $P = \{-3, -1, 1.2, 4\}$ and $C = \{-2,0,3\}$ calculate $R(f, P, C)$.

4. Use areas to calculate $\int_{-3}^{5}(x-2)dx$and$\int_{-3}^{5}|x-2|\,dx$.

5. Use geometry to find$\int_{0}^{t} ax\,dx$.

6. Suppose $\int_{-8}^{3} f(x)dx = 7,\ \ \int_{0}^{3} f(x)dx = 2,\ \ and \int_{0}^{3} g(x)dx = 41.$ Find the following:

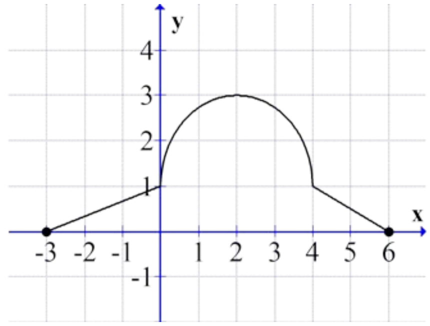
(a) $\int_{-8}^{0} f(x)dx =$

(b) $\int_{0}^{3}(g(x) - f(x))dx =$

(c) $\int_{0}^{3}[12f(x) - 3g(x)]dx =$

(d) $\int_{3}^{-8} f(x)dx =$

7. Consider the sketched function f:
The arc is a semicircle. Compute the following:



(a) $\int_{-3}^{0} f(x)dx =$

(b) $\int_{0}^{4} f(x)dx =$

(c) $\int_{2}^{6} f(x)dx =$

(d) If the area under f from -2 to 6 is estimated using approximating rectangles, find

$L_4 =$ $R_4 =$

8. Properties: Given $\int_{1}^{8} f(x)dx = -4$ and $\int_{-12}^{1} f(x)dx = 6$ and $\int_{1}^{4} f(x)dx = 7$ find

a) $\int_{1}^{8} 4f(x)dx =$

(b) $\int_{4}^{8} f(x)dx =$

c) $\int_{1}^{-12} f(x)dx =$

(d) $\int_{-12}^{8} 3f(x)dx =$

 Understand the methods so you can solve similar problems.
Understand the concepts so you can solve unfamiliar problems.

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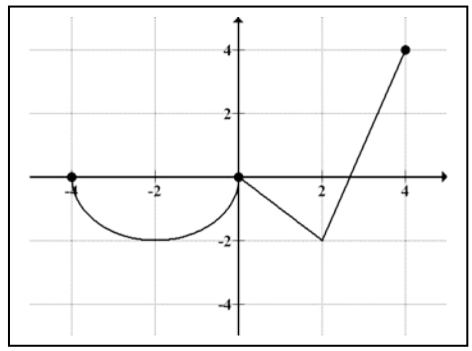
Mathematics 126 Fourth 5.2

1. Sketch the appropriate region and use geometry to calculate the integral

$$\int_0^5 \sqrt{25 - x^2}\, dx$$

$$\int_2^{10} |x - 7| dx$$



2. For the function $f(x)$ as graphed consisting of a semicircle and line segments, find the following:

$$\int_{-4}^0 f(x)dx$$

$$\int_{-2}^2 f(x)dx$$

$$\int_0^4 f(x)dx$$

3. Calculate the following integrals

$$\int_{-1}^2 (3x^2 - 2x + 1)dx$$

$$\int_{-1}^1 (1 + |x^3|)dx$$

 Understand the methods so you can solve similar problems.
Understand the concepts so you can solve unfamiliar problems.

Study the (a) class notes, (b) do the 4th hour problems, (c) study the text examples, (d) do the text exercises and (e) read the next text section.

5.3 Evaluating the definite integral

FUNDAMENTAL THEOREM OF CALCULUS: If a function *f* is continuous on the closed interval *[a,b]* and *F* is any antiderivative of *f*, then $\int_a^b f(x)dx = F(b) - F(a)$.

1. $\int_1^2 e^{3x} dx =$

2. Find the area under $y = 8 - x^2$ from 0 to 2.

3. $\int_{-5}^{3} \frac{dx}{x}$

4. $\int_1^2 \frac{x^2+5x-\sqrt{x}}{x} dx$

5. $\int_1^4 \frac{dt}{t^{1/3}}$

 Understand the methods so you can solve similar problems.
Understand the concepts so you can solve unfamiliar problems.

Study the (a) class notes, (b) do the 4th hour problems, (c) study the text examples, (d) do the text exercises and (e) read the next text section.

Indefinite Integrals, or Antiderivatives

If $F'(x) = f(x)$ for all x in the domain of F then $\int f(x)dx = F(x) + C$.

$$\int \sec^2(x)dx$$

$$\int \frac{dx}{\sqrt{1-x^2}}$$

$$\int \sec(x)\tan(x)dx$$

$$\int \frac{dt}{t^5}$$

$$\int \frac{dx}{\sqrt{x}}$$

$$\int x^n dx$$

Mathematics 126 Fourth 5.3

1. **Use geometric properties** to evaluate the following integrals:

a) $\int_{-2}^{2}\left(4-\sqrt{4-x^2}\right)dx$

b) $\int_{0}^{6}(2x+1)dx$

2. Evaluate the following integrals using the Fundamental Theorem of Calculus:

a) $\int_{0}^{\pi}\left(5x^2+\cos(x)\right)dx$

b) $\int_{0}^{0.5}\left(\frac{2}{\sqrt{1-x^2}}+e^{-3x}\right)dx$

3. Find the indefinite integrals:

a) $\int(2+t^3)(2t-5)dt$

b) $\int\left(x\sqrt[3]{x}+\sec^3(x)\frac{\tan(x)}{1+\tan^2(x)}\right)dx$

 Understand the methods so you can solve similar problems.
Understand the concepts so you can solve unfamiliar problems.

Study the (a) class notes, (b) do the 4th hour problems, (c) study the text examples, (d) do the text exercises and (e) read the next text section.

5.4 The Fundamental Theorem of Calculus Part 2

$$\frac{d}{dx}\int_a^x f(t)dt = f(x)$$

1. $\frac{d}{dx}\int_0^{x} \frac{\sin(t)}{1+t^4} dt =$

2. Express the antiderivative $F(x)$ of $f(x) = e^{\sqrt{x}}$ satisfying $F(23) = -7$ as an integral.

The Chain Rule gives $\frac{d}{dx}\int_c^{a(x)} f(t)dt = f\big(a(x)\big)a'(x)$.

3. Given $g(x) = \int_2^{x^2} \frac{1-t}{1+t} dt$ find $g'(x)$.

4. Find $\frac{d}{dx}\int_{\cos(x)}^{3x^5} \tan(t)\, dt$.

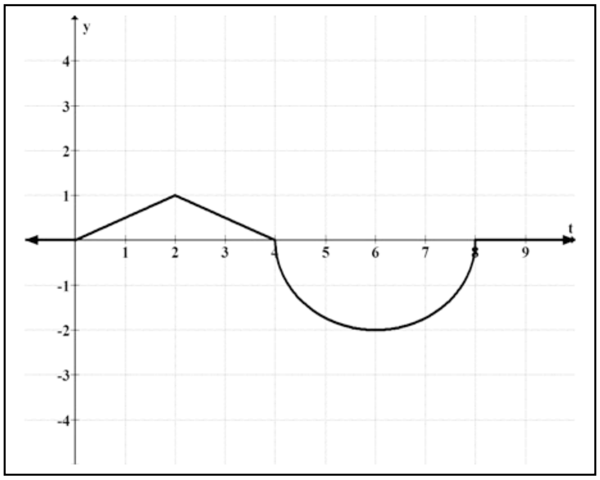
The **net area** between a curve and the *x*-axis is the area above the axis *minus* the area below and equals $\int_a^b f(x)dx$.
The **area** is the area above *plus* the area below, and equals $\int_a^b |f(x)|dx$.

5. For $f(x) = 12 - 3x$ find the net area between $f(x)$ and the *x*-axis on the interval [-3, 12].

6. For $f(x) = 12 - 3x$ find the area between $f(x)$ and the *x*-axis on the interval [-3, 12].

Study the (a) class notes, (b) do the 4th hour problems, (c) study the text examples, (d) do the text exercises and (e) read the next text section.

7. Define $G(x) = \int_0^x f(t)dt$where $f(t)$, composed of line segments and a semicircle, is defined by the graph below.



Find $G(2) = G'(2) =$

$G'(6) = G(9) =$

and $G(0) =$

The maximum value of G is ____________ and the minimum is ___________________.

8. Again using $f(t)$ above, define $J(x) = \int_2^x |f(t)|dt$.

Find $J(0) = J(4) =$

and $J(8) = J'(6) =$

The maximum value of J is ____________ and the minimum is ___________________.

9. Define $F(x) = \int_{-3}^{x} (t-1)(t+2)dt,\ \ x \geq -3.$

a) Find the intervals of increase of F.

b) Find the local minimum(s) and local maximum(s) of F.

c) Find the intervals on which F is concave upward.

10. Define $r(x) = \int_{x/2-4}^{16} (t^3 + e^t)dt$. [Observe $x/2 - 4 \neq \frac{x}{-2}$.] Find the following exactly:

a) $r(8)$

b) $r'(8)$

c) $\frac{d}{dx} r(12)$

11. Find a function f and a number $a > 0$ such that $5 - \int_{a}^{x} \frac{f(t)}{t+1} dt = x^3$.

 Understand the methods so you can solve similar problems.
Understand the concepts so you can solve unfamiliar problems.

Study the (a) class notes, (b) do the 4th hour problems, (c) study the text examples, (d) do the text exercises and (e) read the next text section.

Mathematics 126 **Fourth 5.4**

Find the derivatives of the following functions of x.

1. $\frac{d}{dx}\int_{3}^{2x} \cos(5t)\, dt$

2. $\frac{d}{dx}\int_{\sin(x)}^{x^2} 3r dr$

3. $\frac{d}{dx}\int_{-3}^{\tan(x)} sin(t^3) dt$

4. Find $F'(2)$ for $F(x) = \int_{4}^{2x-7} \frac{t-3}{t^2+2} dt$

5. $\frac{d}{dx}\int_{2x}^{\sin(x)} (16r^2 + 1) dr$

6. Find a function F with derivative $xsin(x)$and $F(12) = 88$.

 Understand the methods so you can solve similar problems.
Understand the concepts so you can solve unfamiliar problems.

Study the (a) class notes, (b) do the 4th hour problems, (c) study the text examples, (d) do the text exercises and (e) read the next text section.

5.5 Net change as the integral of a rate

Applications: If $F' = f$ then $\int_a^b f(x)dx = F(b) - F(a)$ so

$$\int_a^b F'(x)dx = F(b) - F(a)$$

is the total change of F from a to b.

1. Water flows into a reservoir at a rate of $2t^3 - t$ m^3/sec after t seconds. How much water flows into the reservoir from $t = 1$ to $t = 3$ seconds?

2. A population grows at the rate of $\dfrac{100}{1+t^2}$ people per month. Find the population increase, to the nearest person, from month 2 to month 10.

3. An object has acceleration $a(t) = 5 - t$ m/s^2 and initial velocity of 2 m/sec. Find the total distance travelled from $t = 2$ to $t = 20$ seconds. Give your answer to one decimal place accuracy.

4. A particle moving in a straight line has velocity $v(t) = 2\cos(t)$ m/s.
(a) Find the displacement from times 0 s to π s.

(b) Find the distance travelled during [0,π] s.

Repeat using fnInt on the calculator.

5. A particle accelerates at the rate $a(t) = 4 - t^2$ m/s^2 for the first 2 seconds. Estimate the distance travelled over this period if the initial velocity is 3 m/s. Give your answer to 1 decimal accuracy.

Omit Total versus Marginal Cost on page 325.

Mathematics 126, 5.5 Fourth

1. If the acceleration of an object is $a(t) = 3t^2 - 5t$ m/sec, what is the net change in velocity over the interval [1,5] seconds?

2. For the object in question 1, what is the displacement over the interval [1, 5] seconds if the initial velocity is 3 m/s? Does the initial position make a difference?

5.6 Integration by Substitution

Indefinite Integrals: $\int f\big(u(x)\big)u'(x)dx = \int f(u)du$

1. $\int (x^4+5)^{16}x^3dx$

2. $\int sec^2(1-8x)\,dx$

3. $\int \cos(ax+b)\,dx$

4. $\int \frac{\cos(t)}{sin^5(t)}dt$

5. $\int \frac{\sqrt{\theta}}{1-\sqrt{\theta}}d\theta$

 Understand the methods so you can solve similar problems.
Understand the concepts so you can solve unfamiliar problems.

Study the (a) class notes, (b) do the 4th hour problems, (c) study the text examples, (d) do the text exercises and (e) read the next text section.

6. $\int cot^3(\theta)csc^2(\theta)d\theta$

7. $\int \tan(x)\, dx$

Definite Integrals: $\int_{a}^{b} f\big(u(x)\big)u'(x)dx = \int_{u(a)}^{u(b)} f(u)du$

1. $\int_{1}^{9} \frac{e^{\sqrt{x}}}{\sqrt{x}}dx$

2. $\int_{1}^{4} \frac{ln(x^2)}{x}dx$

 Understand the methods so you can solve similar problems.
Understand the concepts so you can solve unfamiliar problems.

Study the (a) class notes, (b) do the 4th hour problems, (c) study the text examples, (d) do the text exercises and (e) read the next text section.

3. $\int_{1}^{6} \frac{2x+1}{\sqrt{x+3}} dx$

4. $\int_{-6}^{0} (3x+2)\sqrt{36-x^2} dx$

 Understand the methods so you can solve similar problems.
Understand the concepts so you can solve unfamiliar problems.

Study the (a) class notes, (b) do the 4th hour problems, (c) study the text examples, (d) do the text exercises and (e) read the next text section.

Math 126 Fourth 5.6, Substitution

Evaluate the integrals:

1. $\int \frac{ln(x^2)}{x} dx$

2. $\int \frac{(\ln(x))^2}{x} dx$

3. $\int tan^4(x) sec^2(x)\, dx$

4. $\int_0^3 x\sqrt{4-x}\, dx$

5. $\int_{-8}^8 x^5 e^{x^2} dx$

6. $\int_0^{\sqrt{2}} x\sqrt{4-x^4}\, dx$

7. $\int_0^1 \frac{sin^{-1}(x)}{\sqrt{1-x^2}} dx$

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Study the (a) class notes, (b) do the 4th hour problems, (c) study the text examples, (d) do the text exercises and (e) read the next text section.

5.7 Further Transcendental Functions

$$\int \frac{dx}{\sqrt{1-x^2}} = sin^{-1}(x) + C, \quad \int \frac{dx}{1+x^2} = tan^{-1}(x) + C, \quad \int \frac{dx}{x} = \ln|x| + C$$

$$\int a^x dx = \frac{a^x}{\ln(a)} + C$$

We omit arcsecant, so omit Example 2.

1. $\int_0^6 \frac{dx}{9x^2+25}$

2. $\int \frac{dt}{\sqrt{5-t^2}}$

3. $\int \frac{3x}{x^4+1} dx$

4. $\int_{-3}^{7} 2^x e^x dx$

5. $\int \frac{dx}{\sqrt{16-3x^2}}$ [Factor 16. Substitute $u = \frac{\sqrt{3}}{4}x$.Use $\frac{d}{du}(sin^{-1}(u)) = \frac{1}{\sqrt{1-u^2}}$.]

6.. $\int \frac{e^{\cos(x)}}{\csc(x)} dx$

Mathematics 126 **Fourth 5.7**

1. $\int \frac{x+7}{\sqrt{36-x^2}} dx$

2. $\int 3^x e^{8x} dx$

3. $\int \frac{dx}{5+7x^2}$

4. $\int \frac{sin^{-1}(e^x)}{\sqrt{1-e^{2x}}} e^x dx$

5. $\int (3x^2 + 1)\sqrt{1 - 2x}\, dx$

6. Suppose $b > 1$. Find all numbers $a < 1$, in terms of b, such that $\int_a^b \frac{\ln(x)}{x} dx = 0$.

 Understand the methods so you can solve similar problems.
Understand the concepts so you can solve unfamiliar problems.

Study the (a) class notes, (b) do the 4th hour problems, (c) study the text examples, (d) do the text exercises and (e) read the next text section.

Mathematics 126
Calculus 2

Chapter 6

Applications

area A = $\int_a^b (upper - lower)dx$

volume Known cross-sectional area: V = $\int_d^e A(y)dy$, $V = \int_a^b A(x)dx$

Disks: V = $\pi \int_a^b [(R(x))^2 - (r(x))^2]dx$

Cylindrical shells: V =2π $\int_a^b p(x)h(x)dx$

arc length L = $\int_a^b \sqrt{\left(\frac{dx}{dy}\right)^2 + 1}\ dy$

work W = $\int_a^b f(x)dx$ or W = FD

 Understand the methods so you can solve similar problems.
Understand the concepts so you can solve unfamiliar problems.

Study the (a) class notes, (b) do the 4th hour problems, (c) study the text examples, (d) do the text exercises and (e) read the next text section.

6.1Area of a Region between Curves

Area is always greater than or equal to 0.

*Definition of area:*Assume $f and g$ are continuous functions with

$$f(x) \geq g(x) for all x in [a, b].$$

Then

$$f(x) - g(x) \geq 0 \; for all x in [a, b].$$

Partition the interval [a, b] into n subintervals of width $\Delta x = \frac{b-a}{n}$ with endpoints

$$x_i = a + i\Delta x for i = 0, 1, 2, \dots, n.$$

The area of a rectangle is

$$\big(f(x_i) - g(x_i)\big)\Delta x.$$

The area of the region between the curves from a to b is approximately

$$Area \approx \sum_{i=1}^{n} \big(f(x_i) - g(x_i)\big)\Delta x.$$

As $n \to \infty$, f *and* g continuous implies the sum tends to an integral, and so to the area, so

$$Area = \lim_{n\to\infty}\left(\sum_{i=1}^{n}\big(f(x_i) - g(x_i)\big)\Delta x\right) = \int_a^b \big(f(x) - g(x)\big)dx$$

Upper minus Lower

First sketch the region and then find the area of the region bounded by the following:

1. $y = x, \;\; y = e^x, \; x = -1, \; x = 2.$

2. $y = x^3 + 2x^2 - 3, \quad y = 3x^2 + 6x - 3.$

 Understand the methods so you can solve similar problems.
Understand the concepts so you can solve unfamiliar problems.

Study the (a) class notes, (b) do the 4th hour problems, (c) study the text examples, (d) do the text exercises and (e) read the next text section.

3. The x-axis and $y = x^3 + 2x^2 - 3x$.

4. Symmetry. $x = -1,\ x = 1,\ y = x^{1/3}$ and the x-axis.

Horizontal rectangles are used when x is a function of y. If $f(y) \geq g(y)\ for\ all\ y\ in\ [c, d]$ then

$$Area = \int_c^d (f(y) - g(y))dy$$

Right minus Left

5. $3y - x = 6,\ x + y = -2,\ x + y^2 = 4.$

 Understand the methods so you can solve similar problems.
Understand the concepts so you can solve unfamiliar problems.

Study the (a) class notes, (b) do the 4th hour problems, (c) study the text examples, (d) do the text exercises and (e) read the next text section.

6. Give integral expressions for the area of the region bounded by

$$y = \ln(x), \qquad y = \frac{1-x}{x}, and\ x = 2$$

(a) using *x* as the variable of integration;

(b) using *y* as the variable of integration.

Find integral expressions for the area of the region bounded by the curves below:

7. $x^{1/2} + y^{1/2} = \sqrt{2}, \ x^2 + y^2 = 4.$

8. $x = |y - 3| + 2, \ y = 5x - 10.$

Mathematics 126 Fourth 6.1, Area

1. Set up an integral expression without absolute values for the exact area enclosed by the parabolas $y = x^2 - 4x + 3$ and $y = 3 - x^2/3$. Approximate with fnInt.

2. Set up an integral expression without absolute values for the exact area of the region enclosed by the curves $x + y^2 = 3$ and $2y = x$. Approximate with fnInt.

3. Set up an integral expression without absolute values for the exact area of the region enclosed by the curves $x = 1.5$, $x = \frac{1}{\sqrt{1-y^2}}$ .Approximate with fnInt.

 Understand the methods so you can solve similar problems.
Understand the concepts so you can solve unfamiliar problems.

Study the (a) class notes, (b) do the 4th hour problems, (c) study the text examples, (d) do the text exercises and (e) read the next text section.

6.2Volume and Average Value

In this section of the text omit Density on pp 367 to 369.

Volumes of known cross sectional area

If a solid *S* has a base region *A*defined for *x* in *[a, b]*and the cross-sections of the solid taken perpendicular to the *x-axis* are *A(x),* then small portions of the solid have volume approximately $A(x_i)\Delta x$ after partitioning the *x-axis* in the usual way. Consequently the volume of the solid can be approximated as

$$V(S) \approx \sum_{i=1}^{n} A(x_i)\Delta x$$

and at the limit the volume is

$$V(S) = \int_a^b A(x)dx$$

.

A similar formula applies if the cross-sections are parallel to the *y-axis*.

1. Set up an integral for the volume of a solid whose base is the ellipse $\frac{x^2}{9} + \frac{y^2}{16} = 1$ and whose cross-sections perpendicular to the *x-axis* are semicircles.

2. Set up an integral for the volume of a solid whose base is the region *A* bounded by the *y-axis*, $y = 2e^{x-6}$, $y = \dfrac{14}{x+1}$ and whose cross-sections perpendicular to the *y-axis* are equilateral triangles. [The area of an equilateral triangle of side *s* is $= \frac{s^2\sqrt{3}}{4}$.]

For the following, provide an appropriate sketch, set up the integral for the exact value, and approximate to 3 decimal accuracy using fnInt.

3. Find the volume of liquid needed to fill a sphere of radius 7 m to a height of 4 m.

Study the (a) class notes, (b) do the 4th hour problems, (c) study the text examples, (d) do the text exercises and (e) read the next text section.

4. Find the volume of the solid bounded by the 3 coordinate planes and the plane through the points (3,0,0), (0,7,0) and (0,0,2).

$$\textbf{Average value} = \bar{f} = \frac{1}{b-a}\int_a^b f(x)\,dx$$

5. Find the average value of $f(x) = \sqrt{1+x}$ on [0, 8].

6. Find the average value of the distance of all points on the curve

$$f(x) = 1 + x^2$$

for $0 \le x \le 3$ from the point (0,5). Give your answer to 3 decimal place accuracy.

Mean Value Theorem for Integrals

If f is continuous on $[a, b]$ then there is a value $c \in [a, b]$ such that

$$f(c) = \frac{1}{b-a}\int_a^b f(x)dx.$$

7. Find c such that $\cos(c) = \dfrac{1}{\pi}\displaystyle\int_{-\pi/2}^{\pi/2}\cos(x)dx$.

8. Find the c guaranteed by the MVTI for $f(x) = \frac{1}{1+x^2}$ on $[0,\ 1\,]$.

 Understand the methods so you can solve similar problems.
Understand the concepts so you can solve unfamiliar problems.

Study the (a) class notes, (b) do the 4th hour problems, (c) study the text examples, (d) do the text exercises and (e) read the next text section.

Mathematics 126 Fourth 6.2

1. Find the volume of a solid remaining after boring a cylindrical hole of radius 2 m out of a sphere of radius 5 m.

2. Find the volume of a solid whose base is an equilateral triangle of side 5 m and whose cross sections parallel to one side of the triangle are semicircles.

3. Find the average value of $f(x) = \sqrt[3]{x}$ on $[0,10]$.

4. Find the point(s) at which the value of $f(x) = \frac{e^x}{r}, r > 0$ equals the average value of the function on the interval $[0, r]$.

 Understand the methods so you can solve similar problems.
Understand the concepts so you can solve unfamiliar problems.

Study the (a) class notes, (b) do the 4th hour problems, (c) study the text examples, (d) do the text exercises and (e) read the next text section.

6.3 Volumes of revolution: Disk method

If a region in the plane is rotated about a line, the resulting solid is called a *solid of revolution* and the line is called*the axis of revolution*.

Disks/Washers
Suppose the region *A*in the plane is bounded by $y = f(x)$, $x = a$, $x = b$ and the x-axis. If *A*is rotated about the x-axis a solid S is formed.

To find the volume of S, partition $[a, b]$, using $\Delta x = \dfrac{b-a}{n}$, where n is a positive integer.

Let $x_i = a + i\,\Delta x$ for $i = 0,1,\ldots,n$. Then *A*is approximated with rectangles, each with base $x_i - x_{i-1}$ and height $f(x_i)$.

As we rotate each rectangle about the x-axis, we generate cylinders with volume

$$V = \pi R^2 h = \pi\left(f\left(x_i\right)\right)^2 \Delta x.$$

Adding, we find the volume of S,

$$V(S) \approx \sum_{i=1}^{n} \pi\left(f\left(x_i\right)\right)^2 \Delta x.$$

If f is continuous, the sum converges as n tends to infinity, and the sum converges to the integral, so

$$V(S) = \pi \int_a^b \left(f(x)\right)^2 dx.$$

Examples
1. The region *A*is bounded by $y = e^x$, *the x- and y-axes and* $x = 2$. Sketch *A*.
Set up integrals (but do not evaluate) for the volume of the solid obtained by rotating *A*about the following:
(a) the x-axis

(b) the y-axis

Washers: More generally, if the radius of the solid is *R(x)* then the volume is $V(S)=\pi\int_a^b (R(x))^2\,dx$ .If the region *A* has an outer radius *R(x)* and an inner radius *r(x)* then the volume is

$$V(S)=\pi\int_a^b \left[(R(x))^2-(r(x))^2\right]dx$$

2. The region *A*is bounded by $y = e^x$, $y = 1$, $x = 2$. Sketch *A*.
Set up integrals (but do not evaluate) for the volume of the solid obtained by rotating *A*about the following:
(a) x = -1;

(b) y = 12.

3. The region A bounded by the x-axis, y = cos(x), y = sin(x), $0 \le x \le \pi/2$. Sketch A.
Set up integrals (but do not evaluate) for the volume of the solid obtained by rotating *A*about the following:
(a) x = 6;

(b) $y = -3$.

4. The region A bounded by the y-axis, $y = 2e^{x-6}$, $y = \dfrac{14}{x+1}$. Sketch A.

Set up integrals (but do not evaluate) for the volume of the solid obtained by rotating *A*about the following:

(a) $x = 21$;

(b) $y = 15$.

 Understand the methods so you can solve similar problems.
Understand the concepts so you can solve unfamiliar problems.

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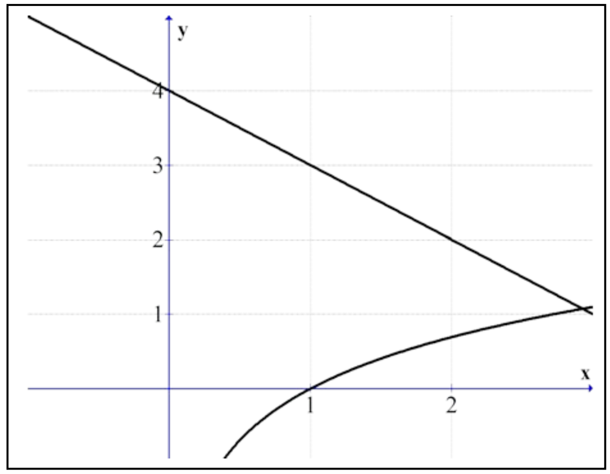
Study the (a) class notes, (b) do the 4th hour problems, (c) study the text examples, (d) do the text exercises and (e) read the next text section.

Mathematics 126 Fourth 6.3, Volume by disks

Consider the region **R** bounded by the x-axis, y-axis, $y = \ln(x)$, $y = -x + e + 1$. Set up an integral expression (but do not evaluate) without absolute values to give the exact volume of the solid:

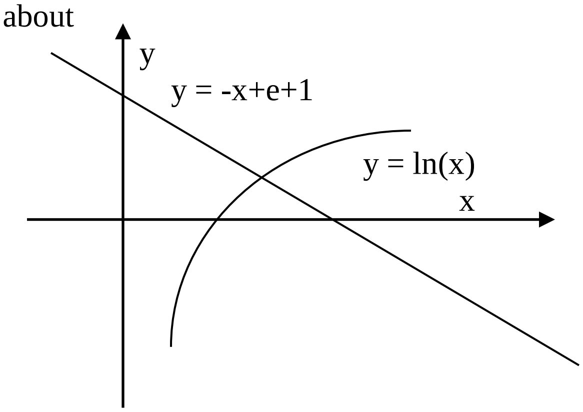
1.a) with base **R** and whose cross sectional areas parallel to the x-axis are equilateral triangles;



(b) with base **R** and whose cross sectional areas parallel to the x-axis are squares;

2. Volume of the solid obtained by rotating **R** about

(a) $x = -1$



(b) $y = 5$

3. Volume of the solid whose base is **R** and whose cross-sections perpendicular to the x-axis are semicircles.

 Understand the methods so you can solve similar problems.
Understand the concepts so you can solve unfamiliar problems.

Study the (a) class notes, (b) do the 4^{th} hour problems, (c) study the text examples, (d) do the text exercises and (e) read the next text section.

6.4 Volume by cylindrical shells

If a region is rotated about a vertical line to create a solid S, the volume can be calculated by taking a cut parallel to the line, of height *h(x)* and at distance *p(x)* from the line. Rotating this cut about the axis of revolution yields a cylindrical shell. In this case the volume is

$$V(S) = 2\pi \int_a^b p(x)h(x)dx$$

1. Use cylindrical shells to set up an integral for the volume of the solid obtained by rotating the region bounded by $y = x^3$ and $y=x^2$, $0 \le x \le 1$ about the following:
a) y = 3;

b) x = -12.

Observe when the method of washers is best, and when the method of shells is best:

2. Set up an integral expression for the exact volume of the solid S obtained by rotating the region R bounded by $f(x) = 3x - 4$ and $g(x) = -x^2 + 2x + 8$ about the line
(a) x = 10;

(b) y = 100.

3. Set up an integral expression for the exact volume of the solid S obtained by rotating the region R bounded by the ellipse $\frac{x^2}{16}+\frac{y^2}{25}=1$ and above the line $y=a, 0<a<5$

(a) about the x-axis;

(b) about the line $x = c$, $c > 5$.

Study the (a) class notes, (b) do the 4th hour problems, (c) study the text examples, (d) do the text exercises and (e) read the next text section.

Mathematics 126, Fourth 6.4, Volume by cylindrical shells

1. Set up an integral expression using shells for the exact volume of a solid remaining after drilling a hole of radius 2 through the centre of a sphere of radius 5. Approximate with fnInt.

2. Set up an integral expression using shells for the exact volume of the solid obtained by rotating the region above the x axis and bounded by $y = x$, $y = -x$, $y = 5 - x^2$ about the line shown below.

(a) $y = 25$. Approximate with fnInt.

(b) $y = a$, $-4 < a < 0$.

 Understand the methods so you can solve similar problems.
Understand the concepts so you can solve unfamiliar problems.

Study the (a) class notes, (b) do the 4th hour problems, (c) study the text examples, (d) do the text exercises and (e) read the next text section.

6.5 WORK

If an object is moved a distance D by a constant force F then the work done is calculated

$$\mathbf{W = FD}$$

For example, to lift a 20 pound suitcase a height of 3 feet, W = 3 (20) = 60 ft-lb, noting that pound is a unit of force, and is abbreviated lb.

For a metric example, to lift a 2 kg stone a height of 1.2 m, note that kg is a measure of mass, not force.

We first calculate the force:
F = ma, where a is the acceleration due to gravity, taken as 9.8 m/s^2
Thus F = 2 (9.8) = 19.6 N, where N denotes newtons of force.

Consequently the work done is W = FD = 19.6 (1.2) = 23.52 J, where J denotes joules of energy and is a product of newton - metres.

The problem occurs when we try to figure out the work required when the distance varies. For example, in pumping out a tank to a fixed height the distance from the bottom is different than the distance from the middle of the tank.

The solution is to break the work up into small bits, each of which can be approximated by a constant W = FD, and then sum the bits, and take the limit, which yields an integral.

Suppose the force f(x) varies as x moves from a to b. Partition [a, b], and observe that on a small subinterval, $W_i \approx f(x_i^*)\,\Delta x$

so $W \approx \sum_{i=1}^{n} f(x_i^*)\,\Delta x$

and taking the limit as $n \to \infty$, we have $\boxed{W = \int_a^b f(x)dx}$

 Understand the methods so you can solve similar problems.
Understand the concepts so you can solve unfamiliar problems.
Study the (a) class notes, (b) do the 4th hour problems, (c) study the text examples, (d) do the text exercises and (e) read the next text section.

Example A force of f(x) = $\sqrt{2+x}$ lb is applied in moving an object from x = 2 to x = 14 ft. How much work is performed?

Hooke's Law says the force F required to maintain a spring x units *beyond its natural length* is F = kx, where k is the positive *spring constant*.

Example A force of 15 N is required to maintain a spring stretched from its natural length of 5 cm to 8 cm. How much work is done in stretching the spring from 8 to 12 cm?

Solution: 1. Find k

2. So F(x) = kx = N

3. Find W = $\int_a^b f(x)dx$ = J

 Understand the methods so you can solve similar problems.
Understand the concepts so you can solve unfamiliar problems.

Study the (a) class notes, (b) do the 4th hour problems, (c) study the text examples, (d) do the text exercises and (e) read the next text section.

PUMPING FLUIDS

In calculating work in metric units, a useful method follows:

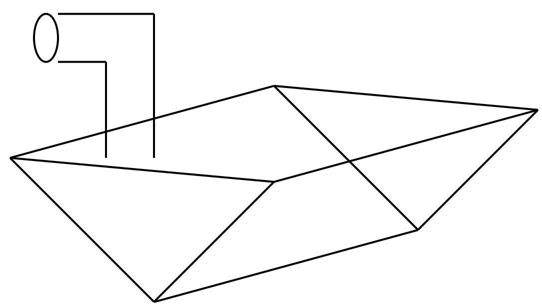
Use these equations:

Mass m = (density) volume

Force f = (mass) gravity (and use 9.8 for the gravitational constant)

Work w = (force) distance

Example: A tank in the shape of a cylinder of length 10 m whose vertical cross sections are isosceles triangles of height 8 m and base 3 m is buried 1 m below ground and filled with a fluid of density 250 kg/m^3. Find the work required to pump all the fluid out of the tank.



Solution: Here we have a variable volume, and a variable distance.

1. Calculate V_i for a horizontal slice of height Δy, length 10 m, and width dependent on y.

2. Then the mass for this slice is $m_i = V_i$ (density)

3. The force $f_i = m_i\, g$

4. The work $W_i = f_i\, d_i$ where the distance d_i depends on the height of the slab.

5. Now W = $\lim\left(\sum W_i\right)$ which can be evaluated as a definite integral.

In the following, provide an appropriate sketch and set up an integral for the exact value of the work. Approximating with fnInt is optional.

1. A container whose horizontal cross-sections are square has a base with sides 40 cm and top with sides 60 cm and is 50 cm high. The container is filled with a liquid of density 1.5 g / cm^3, which is to be pumped out a pipe 30 cm above the top of the container. Find the work required to pump out all the liquid using the x-axis along the top of the container.

2. A pool with triangular cross sections is 8 ft at the deep end, 0 ft at the shallow end, 12 ft wide, 20 ft long. The pool is filled with water (weighing 62.4 lb/ft^3). Set up an integral which represents the work required to pump all the water out of the pool.

3. A tank is obtained by rotating the curve $y = 1 + \sin(x - \pi/2)$, $0 \le x \le \pi$, about the y-axis. y is measured in meters. The tank is full of fluid of density p kg/m^3. Set up a definite integral for the work required to empty the tank by pumping the fluid to the top.

4. A hemispherical tank of radius 5 ft sits on the ground as the bottom half of a sphere. The tank is filled to a depth of 4 ft with oil of weight 55 lbs/ft^3. Taking the x-axis along the bottom of the tank, set up an integral expression for the work required to pump the oil to a pipe 7 ft above the top of the tank.

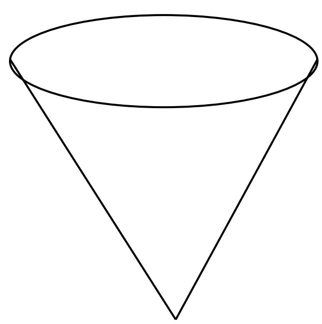
Mathematics 126, Fourth 6.5, Work

1. A spring has a natural length of 0.9 m. A force of 15 N stretches the spring to a length of 1.4 m. Find the work required to stretch the spring 2 m beyond its natural length.

2. An 8 lb bucket is lifted 30 ft up by pulling on a rope that weighs 0.07 lb/ft. (a) How much work is required?

(b) What if the bucket is filled with a fluid weighing 100 lb at the bottom, which leaks at a steady rate so that the fluid weighs 40 lb at the top?

3. A tank is in the shape of an inverted circular cone with height 10 ft and radius 5 ft. The tank is filled with oil weighing 52 lb/ft^3. How much work is done to pump the oil out a valve 6 feet above the top of the tank?



7.1 Integration by parts $\quad \int \boldsymbol{udv} = \boldsymbol{uv} - \int \boldsymbol{vdu}$

L I A T E

1. $\int xe^{2x}\,dx$

2. $\int xln(x)dx$

One term:

3. $\int tan^{-1}(x)\,dx$

4. $\int \ln(x)\,dx$

Repeated application

5. $\int x^2\cos(x)\,dx$

6. $\int x^2e^{3x}dx$

Reversal

7. $\int e^{2x}\cos(5x)\,dx$

Definite Integral: 8. $\int_{-1/2}^{1/2} sin^{-1}(x)dx$

9. $\int_{-1/2}^{0} sin^{-1}(x)dx$

10. $\int_{1}^{e} \sin(\ln(x))\, dx$

Combining u-substitution and IP
11. $\int x^3\, sin(x^2)dx$

12. $\int e^{x^2}\, x^3 dx$

Omit reduction formulas. p.402 and example 7.

Mathematics 126, Fourth 7.1

Evaluate the integrals:

1. $\int t^3 \ln(t)\, dt$

2. $\int e^{-2x} \cos(x)\, dx$

3. $\int_0^{1/2} sin^{-1}(x) dx$

4. $\int x5^x dx$

 Understand the methods so you can solve similar problems.
Understand the concepts so you can solve unfamiliar problems.

Study the (a) class notes, (b) do the 4th hour problems, (c) study the text examples, (d) do the text exercises and (e) read the next text section.

7.2 Trigonometric Integrals

A. Odd powers of sine and cosine

1. $\int cos^4(x)sin^3(x)dx$

2. $\int cos^3(2x)sin^2(2x)dx$

B. Even powers of sine and cosine

$$\boldsymbol{cos^2(t) = \frac{1 + cos(2t)}{2}, \quad sin^2(\oplus) = \frac{1 - cos(2\oplus)}{2}}$$

3. $\int sin^4(3x)dx$

4. $\int cos^4(5t + 7)dt$

C. Tangent and secant

$$\int \boldsymbol{tan(t)dt = ln|sec(t)| + C}$$

$$\int \boldsymbol{sec(\boxtimes)d\,\boxtimes = ln|sec(\boxtimes) + tan(\boxtimes)| + C}$$

$$\boldsymbol{tan^2(x) = sec^2(x) - 1}$$

5. $\int tan^2(3t)dt$

6. $\int tan^5(3x+1)sec^2(3x+1)dx$

7. $\int (tan^2(8x) - \sec(3x))dx$

Omit reduction formulas, examples 3, 4, 6,7, 8 and *cos(mx)sin(nx)* and the table of Trig Integrals on p. 410.

Mathematics 126 **Fourth 7.2 Trigonometric Integrals**

1. $\int \cos(x)\, sin^4(x) dx$

2. $\int sin^2(3r) cos^2(3r) dr$

3. $\int cos^4(x) sin^7(x) dx$

4. $\int \sec^8(2w+4)\tan(2w+4)\, dw$

 Understand the methods so you can solve similar problems.
Understand the concepts so you can solve unfamiliar problems.

Study the (a) class notes, (b) do the 4th hour problems, (c) study the text examples, (d) do the text exercises and (e) read the next text section.

7.5 Integration by Partial Fractions

Proper fractions: 1. $\int \frac{2x-3}{(2x-5)(x+2)}dx$

Repeated linear factors

2. $\int \frac{x^2-5}{(x-7)^2(x+4)}dx$

Improper fractions: Numerator degree $\geq$ denominator degree

Using long division: 3. $\int \frac{3x^3+9x^2-11x+6}{x^2+3x-4}dx$

 Understand the methods so you can solve similar problems.
Understand the concepts so you can solve unfamiliar problems.

Study the (a) class notes, (b) do the 4th hour problems, (c) study the text examples, (d) do the text exercises and (e) read the next text section.

Use the arctangent for irreducible quadratic factors: $\int \frac{dx}{x^2+a} = \frac{1}{\sqrt{a}}\tan^{-1}\left(\frac{x}{\sqrt{a}}\right)+C$ for $a > 0$.

4. $\int \frac{x-7}{x^3+5x^2+2x+10}dx$

 Understand the methods so you can solve similar problems.
Understand the concepts so you can solve unfamiliar problems.

Study the (a) class notes, (b) do the 4th hour problems, (c) study the text examples, (d) do the text exercises and (e) read the next text section.

Mathematics 126 **Fourth 7.5, Partial Fractions**

Evaluate the integrals:

1. $\int \frac{x+4}{(x-3)(x+7)}\,dx$

2. $\int \frac{x^2-3x+1}{x^3+x^2+9x+9}\,dx$

 Understand the methods so you can solve similar problems.
Understand the concepts so you can solve unfamiliar problems.

Study the (a) class notes, (b) do the 4th hour problems, (c) study the text examples, (d) do the text exercises and (e) read the next text section.

7.6 Improper Integrals

Determine if each of the following integrals is convergent or divergent. If the integral converges, find its value if possible.

A. Infinite limits

1. $\int_1^{\infty} \frac{dx}{x}$

2. $\int_{-\infty}^{0} x\cos(x)dx$

3. $\int_1^{\infty} \frac{dx}{x^2+2x+5}$

 Understand the methods so you can solve similar problems.
Understand the concepts so you can solve unfamiliar problems.

Study the (a) class notes, (b) do the 4^{th} hour problems, (c) study the text examples, (d) do the text exercises and (e) read the next text section.

B. Discontinuities

5. $\int_0^1 \ln(x)dx$

C. More exercises. Determine if each of the following integrals converges or diverges. If possible, evaluate the convergent integrals.

6. $\int_0^\infty \cos(x)dx$

7. $\int_0^1 \frac{1}{\sqrt{x}}dx$

8. $\int_0^1 \frac{1-e^{-10x}}{1-e^{-5x}}dx$ [Hint: you can do some algebra and not need to take limits.]

Mathematics 126 **Fourth 7.6, Improper Integrals**

Evaluate the integrals:

$$\int_{-\infty}^{3} \frac{x}{\left(x^2+9\right)^4}\,dx$$

$$\int_{0}^{1} \frac{\ln(x)}{x}\,dx$$

Find the volume of the solid S obtained by rotating the region R bounded by the function $f(x) = \frac{1}{(x-2)^{3/4}}$ and the x-axis on the interval (2, 3] (a) about the y-axis and (b) about the x-axis. First sketch the region R.

Chapter 8, Further applications of the integral, and Taylor Polynomials

8.1 Arc Length

If f is continuous and differentiable on [a,b] then $s = \int_a^b \sqrt{1 + (f'(x))^2}dx$

Give all approximations to 3 decimal accuracy.

1. Approximate the length of $x^2 - y^2 = 4,\ 3 \le x \le 5, y \ge 0$.

2. Approximate the length of $x \ln(y) = 1$ from (1,e) to (2, $\sqrt{e}$). Set up two integrals for the exact length, on with respect to *x* and the other with respect to *y*.

3. Approximate the length of $f(x) = (1 - x)^{2/3}$ on [-2, 6]. Caution!

 Understand the methods so you can solve similar problems.
Understand the concepts so you can solve unfamiliar problems.

Study the (a) class notes, (b) do the 4th hour problems, (c) study the text examples, (d) do the text exercises and (e) read the next text section.

4. Set up an integral expression for the exact length of the ellipse $\frac{x^2}{16} + \frac{y^2}{25} = 1$.

Observe the tangent line is vertical when x = -4 and x = 4 so we cannot simply solve for y and integrate. Another approach: by symmetry we only need to find the length on ¼ of the curve, say from x = 0 to x = 4 with $y \geq 0$. We can find an integral with respect to x from x = 0 to x = 3, and another with respect to y for the remaining part of the curve. Can you find an even easier approach?

Omit surface area, from the bottom of p 469 to the top of p 471.

Mathematics 126, Fourth 8.1, Arc length

Omit example 2 and surface area following example 3 to the end of the section.
Give all approximations to 3 decimal accuracy.

1. Use fnInt to estimate the length of the arc determined by $y = \frac{x^2}{2}$ from (0, 0) to $\left(\frac{1}{2}, \frac{1}{8}\right)$.

2. Use fnInt to estimate the length of the curve y = 2x + cos(x), $0 \le x \le 2\pi$.

3. Set up an integral expression for the perimeter of the region bounded by $f(x) = 3x^2 - 24x - 27$ and $g(x) = -5x^2 + 40x + 45$.

 Understand the methods so you can solve similar problems.
Understand the concepts so you can solve unfamiliar problems.

Study the (a) class notes, (b) do the 4th hour problems, (c) study the text examples, (d) do the text exercises and (e) read the next text section.

8.4 Taylor Polynomials

Omit the Error Bound from the bottom of p. 492 to the top of p. 495. Do error estimates with the GC.

Linear approximation: $T_1(x) = f(a) + f'(a)(x-a)$

Quadratic approximation: $T_2(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2}(x-a)^2$

Taylor Polynomials

$T_3(x) = T_2(x) + \frac{f^{(3)}(a)}{3!}(x-a)^3$ satisfies $T_3^{(k)}(a) = f^{(k)}(a) for\ k = 0, 1, 2, 3.$

The $\boldsymbol{n^{th}}$ **order Taylor polynomial centered at** $\boldsymbol{x = a}$ is

$$T_n(x) = \sum_{k=0}^{n} \frac{f^{(k)}(a)}{k!}(x-a)^k$$

1. Find the 5th-order Taylor polynomial for $f(x) = \cos(x)\ centered\ at\ x = \frac{\pi}{2}$.

Use the above Taylor polynomial to estimate $cos(1.5)$. Compare with the calculator value. The absolute value of the difference is approximately the *remainder*.

Study the (a) class notes, (b) do the 4th hour problems, (c) study the text examples, (d) do the text exercises and (e) read the next text section.

2. Approximate $\frac{1}{2.1^3}$ with the 4th-order Taylor polynomial for $f(x) = \frac{1}{(x+2)^3}$ centered at $a = 0$.

3. Find the nth -order Taylor polynomial $T_n(x)$ for $f(x) = e^{5x}$ centered at $x = 1$.

5. Approximate $\sqrt[5]{33}$ using an appropriate 3rd –order Taylor polynomial $T_3(x)$.

6. Find the 3rd Taylor polynomial $T_3(x)$ for $f(x) = \ln(2x - 1)$ at $a = 1$. Find the largest interval containing $a = 1$ on which $T_3(x)$ approximates $f(x)$ within 0.01.

Mathematics 126 Fourth 8.4, Taylor polynomials

1. For $f(x) = \frac{1}{2x+1}$ find the following:

a) the third-order Taylor polynomial centered at $a = 0$.

b) use the polynomial to estimate $\frac{1}{1.97}$.

c) the 4^{th}-order Taylor polynomial centered at $a = -1$.

d) use the polynomial to estimate $-\frac{1}{1.04}$.

2. Use a 5^{th}-order Taylor polynomial to estimate $\sqrt[3]{124}$.

 Understand the methods so you can solve similar problems.
Understand the concepts so you can solve unfamiliar problems.

Study the (a) class notes, (b) do the 4^{th} hour problems, (c) study the text examples, (d) do the text exercises and (e) read the next text section.

Chapter 9 Introduction to Differential Equations

9.1 Solving differential equations

Omit the material after example 2 to the end of the section.

Some first order differential equations:

1. Find the general solution to $\frac{dy}{dx} = 5e^{-x}$.

An initial value problem:

2. Find the particular solution to $(y^2 + e^{\pi y})\frac{dy}{dx} = 3x - \frac{5}{x}$ with initial condition $y(1) = 2$.

Some second order differential equations.

3. Verify that $y = e^{-5x} + 2x$ is a solution to the differential equation
$50y + 5y' - y" = 100x + 10$.

4. If $y = e^{wt}$ is a solution to the differential equation $ay" + by' + cy = 0$ then the constant w must satisfy some quadratic equation. Find the quadratic equation.

5. Use the result of the last question to find all solutions involving the natural exponential function to the DE $10y'' + 17y' + 3y = 0$.

Separable differential equations $\frac{dy}{dx} = f(x)g(y)$

1. Solve the equations $\left(1 + x^2\right)y' = \frac{1}{\sqrt{2y+1}}$.

2. $\frac{dy}{dx} = 2^{3y}\tan(x) + x2^{3y}$

Mathematics 126 Fourth 9.1, Differential equations

1.a) By experimenting, find a solution to the differential equation $y'' + y = 0$.

b) Can you, similarly, find a solution to $y'' - y = 0$?

2. Show that one solution to the differential equation $y'' + 9y = 0$ is $y = cos(3x)$.

3. Show that one solution to the differential equation $y'' - 9y = 0$ is $y = e^{3x}$. Can you find other solutions?

4. Solve the initial value problem $\frac{xy}{(5-2x)(x+4)} = (6y^3 + 1)\frac{dy}{dx}$, $y(2) = e^2$. Simplify your answer.

Orthogonal Trajectories

An **orthogonal trajectory** of a family of curves is a curve that intersects each curve of the family at right angles.

1. Find the orthogonal trajectories of the family of circles $x^2 + y^2 = r^2$.
Graph. Observe.

Method:
1. Find the DE describing the family
2. Solve the DE for dy/dx.
3. Write dy/dx for the orthogonal family.
4. Solve the DE

2. Find the orthogonal trajectories of the family $y = ke^{-x}, \quad k \neq 0$.

3. Find the orthogonal trajectories of the family $y^2 = kx^2$.

4. Find the orthogonal trajectories to the family of curves $xy = c, c \neq 0$.

 Understand the methods so you can solve similar problems.
Understand the concepts so you can solve unfamiliar problems.

Study the (a) class notes, (b) do the 4th hour problems, (c) study the text examples, (d) do the text exercises and (e) read the next text section.

Mixture problem

A tank which holds 500 L of orange juice mixture is half full, containing water and 2 kg of pure juice. At 9 am two taps open and out of one tap pours 3 L/min of mixture containing 0.3 kg/L of pure juice, while the other tap produces 7 L/min of mixture containing 0.7 kg/L of pure juice. The mixture is well stirred, thoroughly mixed and leaves the tank at the rate of 10L/min.

a) Set up a differential equation to model this activity, letting y be the amount of pure juice in the tank t minutes after 9 am.

b) Solve the differential equation.

Mathematics 126 Fourth DE, Mixture and Orthogonal trajectories

1. A tank contains 30 kg of dissolved salt in 3000 L of water. Brine containing 0.04 kg salt per L of water is entering the tank at the rate of 20 L/min. The solution is kept thoroughly mixed and drains at the same rate. How much salt is in the tank after an hour? How much salt is in the tank after a very long time?

c) When will the tank contain 100 kg of salt?

2. Find the orthogonal trajectories of the family of curves $3x^2 = ky^2 + 1$ where k is an arbitrary constant.

3. Find the orthogonal trajectories of the family of curves $kx^5 = y^4$ where k is an arbitrary constant.

 Understand the methods so you can solve similar problems.
Understand the concepts so you can solve unfamiliar problems.

Study the (a) class notes, (b) do the 4th hour problems, (c) study the text examples, (d) do the text exercises and (e) read the next text section.

Math 126 Review Question for Chapters 7 to 9

1. $\int (5x^2 - 3x + 2)e^{2x}dx$

2. $\int \frac{3x^3-28x^2-4x-52}{(x+2)(x-4)(x^2+2)}dx$

3. $\int_2^7 \frac{dx}{(x-4)^{1/3}}$

4. $\int_{-\infty}^0 \frac{dx}{1+x^2}$

5. Set up an integral expression for the exact arc length of the hyperbola $x^2 - 5y^2 = 4$ from the point (3,1) to (3,-1). A sketch may help. Approximate to 2 decimal place accuracy.

6. (a) Find the 4th Taylor Polynomial for $f(x) = e^{6-3x}$ centered at $a = 2$.

(b) Find the largest interval containing 2 on which $T_4(x)$ differs from $f(x)$ by at most 0.01.

7. Solve $\sec(t)y\,y' = e^{2t}\sqrt{(y+1)}$

8. A tank contains 300 L of pure water. Brine containing 0.04 kg of salt per L of water enters at the rate of 5 L/minute, is thoroughly mixed, and leaves at the rate of 5 L/minute. Find the amount y of salt in the tank 20 minutes after the process begins.

1. $\frac{e^{2x}}{2}(5x^2 - 8x + 6) + C$, 2. $\ln\left|\frac{(x+2)^5\sqrt{x^2+2}}{(x-4)^3}\right| + C$, 3. $\frac{3}{2}\left(3^{2/3} - 2^{2/3}\right)$, 4. $\frac{\pi}{2}$,
5. $2\int_0^1 \sqrt{1 + \frac{25y^2}{4+5y^2}}\,dy \approx 2.93$,
6(a) $T_4(x) = 1 - 3(x-2) + \frac{9}{2}(x-2)^2 - \frac{9}{2}(x-2)^3 + \frac{27}{8}(x-2)^4$, (b) [1.666, 2.358]
7. $\frac{2}{3}(y+1)^{3/2} - 2\sqrt{y+1} = \frac{2}{5}e^{2t}\left(\cos(t) + \frac{1}{2}\sin(t)\right) + C$, 8. 3.402 kg.

Mathematics 126
Integral Calculus

Chapter 10

Infinite Sequences and Series

Sequences $\mathbf{a_1, a_2, a_3, \ldots}$

Series $S = \sum_{n=1}^{\infty} a_n = \lim_{n \to \infty} s_n$

 Understand the methods so you can solve similar problems.
Understand the concepts so you can solve unfamiliar problems.

Study the (a) class notes, (b) do the 4th hour problems, (c) study the text examples, (d) do the text exercises and (e) read the next text section.

10.1 Sequences

Omit Bounded Sequences pp 543 to 545.

*A **sequence** is an ordered list of numbers* $\{a_1, a_2, a_3,...,a_n,...\}$.
*A **recurrence relation** of the form* $f(a_n) = a_{n+1}$ may define the sequence.
*An **explicit formula** of the form* $f(n) = a_n$ may define the sequence.

1. For the sequence $-\frac{1}{2},\frac{2}{4},-\frac{6}{8},\frac{24}{16},...$ (a) find a recurrence relation defining the sequence and

(b) find an explicit formula for the sequence.

2. Find the first 5 terms of the sequence defined by the recurrence relation

$$a_1 = -2,\ a_{n+1} = 1 + a_n/2$$

Theorem: *If f is a function with $\lim_{x\to\infty} f(x) = L$ and a_n is a sequence such that $f(n) = a_n$ for every positive integer n, then $\lim_{n\to\infty} a_n = L$.*

The limit of the sequence equals the limit of the associated function.

1. Determine if the sequence converges and the limit, or that the sequence diverges:

$$-\frac{1}{2},\frac{2}{4},-\frac{6}{8},\frac{24}{16},...?$$

2. Does the sequence with n^{th} term shown below converge? If so, find the limit.

a) $a_n = (-1)^n \dfrac{n^2+1}{(1+n)(1-n^2)}$

b) $a_n = \dfrac{2n+1}{1-4n}$

c) $a_n = \dfrac{n\sin\left(\frac{8}{n}\right)}{\left(2^n+1\right)}$

d) $a_n = \frac{e^n}{\ln(n)}$

e) $\{(-1)^{2n}\}$

Factorial: Define 0! =1, 1! = 1 and for *n > 1, n! = n (n-1)!*

f) $a_n = \frac{18^n}{n!}$

g) $\{\cos(n\pi)\}$

h) $a_n = \sin(n\pi)$

i) $\left\{\frac{2n+(-1)^{n}}{2n-(-1)^{n}}\right\}$

A **nonincreasing sequence** satisfies $a_n \geq a_{n+1}$ *for all n*. A **nondecreasing sequence** satisfies $a_n \leq a_{n+1}$ *for all n*. A **monotonic sequence** is either nonincreasing or nondecreasing. A **bounded sequence** satisfies $|a_n| \leq$ M for all *n* and some M.

Example The sequence $a_n = \frac{(-1)^n}{12^n}$ in not monotonic, but is bounded because $|a_n| \leq 1$.

Determine if the sequence $a_n = 1 - \frac{5}{n}$ is (a) monotonic,

(b) bounded.

Sequences on the TI83+ and TI84+

The Graphing Calculator has a variety of options for calculating the values of terms in a sequence, and graphing sequences.

Graphing a sequence

1. A calculator will be able to graph only a finite number of terms.
2. In contrast to a list (see next section) , the calculator will easily graph the sequence

$a_{n+1} = \sqrt{2 + a_n}$.

3. To graph a sequence, look in [MODE] [Seq] [Y =]
4. The syntax requires:

nMin =	The smallest value of the subscript n
\u(n) =	The typical sequence term
u(nMin)	The value of the first term

Example To graph the first 50 terms of the sequence

$$a_{n+1} = \sqrt{2 + a_n} \text{ , } n \geq 1, \; a_1 = 3$$

type [ON] [MODE] [Seq]
[Y=]
nMin = [1] [▼]
\u(n) = [2nd] [x^2 $\sqrt{\ }$] [2+] [2nd] [7 u] [(] [x,T,θ,n] [-] [1] [ )] [ )] [▼]
u(nMin) = [3]

[WINDOW]
nMin = [1] [ENTER]
nMax = [50] [ENTER]
PlotStart = [1] [ENTER]
PlotStep = [1] [ENTER]
Xmin = [0] [ENTER]
Xmax = [50] [ENTER]
Xscl = [5] [ENTER]
Ymin = [0] [ENTER]
Ymax = [5] [ENTER]
Yscl = [1] [GRAPH]

To see some of the values, press [TRACE] [▶]. Does the sequence converge?

Exercise: Try different values for a_1, say 1, 5, and see the effect on convergence
See Chapter 6 of the TI83+ manual for further information.

A **geometric sequence** is a sequence in which $\frac{a_{n+1}}{a_n} = r for\ all\ n$. The ratio of succeeding terms is constant and does not depend on n.

Determine if each of the following sequences is geometric. If the sequence is geometric, find the ratio r.

1. $\frac{1}{(-3)^n}$,

2. $(-0.8)^n$,

3. $12\left(\frac{5}{4}\right)^n$

4. $a_n = 1 + \frac{5}{n}$

Theorem: Suppose a geometric sequence has ratio r and first term a.
If $|r| < 1$ the sequence converges to 0.If $|r| \geq 1$ the sequence diverges.

Determine which of the geometric sequences above converge to 0, and which diverge.

Mathematics 126 Fourth 10.1, Sequences

1. Determine the convergence or divergence of the following sequences. If a sequence converges, find the limit: (a) $a_n = \dfrac{5n+2}{1+\sqrt{n}}$

(b) $\{\sin(n)\}$

(c) $a_n = \dfrac{n(\sqrt{n}-1)}{2+5n^{3/2}}$

(d) $\left\{\dfrac{2^n}{(2n+1)!}\right\}$

2. Find a formula for the general term a_n of the sequence
$\left\{-3, \frac{9}{2}, \frac{-27}{6}, \frac{81}{24}, \frac{-243}{120}, \ldots\right\}$

3. Determine which of the following sequences are geometric. Find the limit, if it exists, of each.
a) $a_n = 1/4^n$

b) $a_n = n + \tan(n\pi/2)$

4. For the sequence $\{20, 15, 10, 5, \ldots\}$ find (a) an explicit formula for the sequence and (b) determine if the sequence converges and its limit, or diverges.

Study for the final exam starting now

Work through **all** the tests and sample final exams in the booklet of previous Capilano exams. Time yourself on each test and exam. Check the solutions in the back of the booklet. Discuss in the Math Learning Centre and/or with Frank any concerns you wish.

10.2 Summing an infinite series

A *partial sum* $S_n = a_1 + a_2 + a_3 + \cdots + a_n = \sum_{k=1}^{n} a_k$ is the sum of the first n terms of a sequence. A *series* or *infinite series* is the limit of the partial sums and is written

$$S = \lim_{n\to\infty} S_n = \sum_{n=1}^{\infty} a_n$$

Theorem: If $\sum a_n$ and $\sum b_n$ converge, then $\sum(a_n \pm b_n)$ and $\sum ca_n$ also converge for any constant c, and $\sum a_n \pm \sum b_n = \sum(a_n \pm b_n)$ and $\sum ca_n = c\sum a_n$.

Divergence Test: If $\lim_{n\to\infty} a_n \neq 0$ then $\sum a_n$ diverges.

Determine if each of the following series converges. If the series converges, find the sum, or approximate the sum if you cannot find it exactly. Name the test you use [for example, harmonic series, divergence test, telescoping series].

1. $\displaystyle\sum_{1}^{\infty} \frac{(n-4)^2}{5n(n+2)}$

2. $\sum_{n=0}^{\infty} \frac{3^{n+1}}{3^n+1}$

3. The n^{th} partial sum is $s_n = \dfrac{2n-1}{5n+3}$ (a) Find the n^{th} term a_n.

(b) Find the sum of the series.

The **Harmonic series** $\sum_{1} \frac{1}{n}$ diverges.

4. $\displaystyle\sum_{1}^{\infty} \frac{2}{5n}$

Definition: A series $\sum a_n$ is **geometric** if the ratio $\frac{a_{n+1}}{a_n} = r$ is constant for all n.

Theorem: If a geometric series has ratio r and first term a, then

(i) the series sums to $\frac{a}{1-r}$ if $|r| < 1$or

(ii) diverges if $|r| \geq 1$.

Determine if each of the following series is geometric. For the geometric series, determine if the series converges, in which case find the sum.

5. $0.1 + 0.01 + 0.001 + 0.0001 + \ldots$

6. $\sum_{n=1}^{\infty} \frac{\ln(n)}{2^n}$

7. $\sum_{n=2}^{\infty} \left(\frac{1}{3^n} - \frac{1}{4^n}\right)$

Telescoping Series are series in which the partial sum S_n collapses to a few terms. Find the sum of the following series, if the series converges. If the limits are not stated assume the series sums from n =1 to ∞.

8. $\sum \frac{1}{n(n+1)}$

9. $\sum ln\left(\frac{n+1}{n}\right)$

10. **Rewriting series:** If $\sum_{\mathbf{0}}^{\infty}\left(\frac{\mathbf{2^{n+1}}}{\mathbf{cos(0.8n)}}\right) = \sum_{\mathbf{1}}^{\infty}(\boldsymbol{b_n})$ find $\boldsymbol{b_n}$.

Series on the TI83+ and TI84+

If you understand how to work with sequences on the graphing calculator, series are the natural next step. The GC will calculate partial sums very quickly.

Example Find the first 20 partial sums of the series $\sum_{1}^{\infty} \frac{1}{n!}$.

[ON] [MODE] [Seq] [Y=]
nMin = [1] [ENTER]
u(n) = [1] [/] [X,T,θ,n] [MATH] [◀] [4] [+] [2nd] [7] [(] [X,T,θ,n] –1] [)] [ENTER]
u(nMin) = [1] [WINDOW]

nMin = [1]
nMax = [20]
PlotStart = [1]
PlotStep = [1]
Xmin = [1]
Xmax = [20]
Xscl = [1]
Ymin = [0]
Ymax = [5]
Yscl = [1] [GRAPH]

Now trace and see if you can guess the value to which $\sum_{1}^{\infty} \frac{1}{n!}$ converges!

Mathematics 126 Fourth 10.2, Series

1. Determine if each of the following series is convergent or divergent. State the names of any tests you use. If the series converges, find the sum. Assume the sum is from n = 1 to ∞ if no limits are shown.

(a) $\sum_{1}^{\infty} \frac{3}{n}$ (b) $\sum \frac{\pi^{n+2}}{4^{n-1}}$ (c) $\sum \left(\cos\left(\frac{1}{n}\right) - \cos\left(\frac{1}{n+1}\right) \right)$

(d) $\sum \frac{1-2n}{3n+4}$

2. The nth partial sum of the series $\sum a_n$ is $s_n = \frac{2n+1}{3n-1}$.

Find (a) a_n and (b) the sum of the series.

3. Determine whether the series converges or diverges. State the names of any tests you use. If the series converges find the sum exactly, or if the exact sum is not attainable, estimate the sum to 3 decimal accuracy.

(a) $\sum_{n=1}^{\infty} \left(\sqrt[3]{n+2} - \sqrt[3]{n+1} \right)$

(b) $\sum_{k=3}^{\infty} \left(\frac{-5}{7} \right)^{2k}$

(c) $\sum_{k=2}^{\infty} (0.97 + \sin(k\pi))^{k}$

(d) $\sum_{r=1}^{\infty} \left(\frac{3^r}{r^3} \right)$

4. Use your calculator to find to 5 decimal accuracythe value of the 20th

(a) term of the sequence {sin(n)/n}

(b) partial sum of the series $\sum \frac{2^n}{n!}$.

10.4 Alternating Series Test (Leibniz Test)

Omit Absolute and Conditional Convergence. When the text asks if aseries converges absolutely or converges conditionallywe need only show the seriesconverges.

AST or Leibniz Test: If for some integer M, $a_n > a_{n+1}$ for all $n \geq$ M and $\lim_{n\to\infty} a_n = 0$ then $\sum_1^\infty (-1)^n a_n$ = S converges. Moreover if the series converges to S then

$$|S - S_n| < a_{n+1}$$

1. Estimate the error (to three decimal places) in approximating the sum of the series $\sum_{n=1}^{\infty} \frac{(-1)^n 2^n}{n!}$ by the sum of the first six terms. Is the error less than 0.05? **In your calculator put to find the error, put in u(n) = 2^n/n!, in TABLE read each value as n+1, rather than the n shown at the top of the table.**

2. Determine if the following series converge: (a) $\sum_{n=1}^{\infty} \frac{(-1)^n (n^3-1)}{(n^2+3)(n^2+8)}$

(b) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} \ln(n)}{5\ln(n+1)}$

2. Approximate $\sum_1^{\infty} \frac{(-1)^n}{3^n n!}$ so that the error is less than 10^{-5}.

3. Determine if the series converges. If the alternating series converges, determine the smallest value of n necessary to estimate the sum and be within 0.01 of the exact sum.

a) $\sum_{n=1}^{\infty} \frac{(-1)^{n-3}(6n^2-2n+3)}{(5+n)(2n+7)}$

b) $\sum_{1}^{\infty} (-1)^{n+1} 5e^{-2n}$

Study the (a) class notes, (b) do the 4th hour problems, (c) study the text examples, (d) do the text exercises and (e) read the next text section.

Mathematics 126 Fourth 10.4, Alternating series test

Determine whether the series converges or diverges, stating the test you use. If the series converges, find the exact sum if possible; otherwise find the smallest value of n such that the sum of the first n terms is in error by less than 0.001.

(a) $\sum_{n=1}^{\infty} (-1)^n \frac{3^n}{(2n+1)!}$

(b) $\sum (-1)^n \frac{3n-1}{2n+1}$

(c) $\sum \frac{(-10)^n}{n!}$

 Understand the methods so you can solve similar problems.
Understand the concepts so you can solve unfamiliar problems.

Study the (a) class notes, (b) do the 4th hour problems, (c) study the text examples, (d) do the text exercises and (e) read the next text section.

10.5 The Ratio Test

Omit the Root Test.

Ratio Test: If $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| < 1$ *then* $\sum a_n$ *converges*.

If $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| > 1$ *then* $\sum a_n$ *diverges*.

If $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = 1$ *then the test is inconclusive so use another test*.

1. Determine if the following series converge or diverge.

(a) $\sum_{1}^{\infty}\frac{12^n}{(n+1)5^n}$

(b) $\sum n^4 e^{-n}$

(c) $\sum_{n=1}^{\infty}\frac{\ln(n)}{n!}$

2. Find all values of τ for which the following series converges: $\sum_{k=1}^{\infty}\frac{\tau^k k}{(k+1)^k}$

 Understand the methods so you can solve similar problems.
Understand the concepts so you can solve unfamiliar problems.

Study the (a) class notes, (b) do the 4th hour problems, (c) study the text examples, (d) do the text exercises and (e) read the next text section.

10.5 Fourth, Ratio Test

Determine the convergence or divergence of each of the following series.

1. $\sum(-1)^n \frac{2^n}{n!}$

2. $\sum \frac{n+3}{7^{2n+1}}$

3. $\sum \frac{5^n}{n^5}$

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10.6 Power series

We use the ratio test and set $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| < 1$ and solve for x to determine the open interval of convergence.

1. Find the radius of convergence and the open interval of convergence of $\sum_0^\infty 2^n x^n$.

2. Find the radius of convergence and the open interval of convergence of $\sum_1^\infty \frac{(x+2)^n}{n}$.

3. Find the radius of convergence and the open interval of convergence of $\sum_0^\infty n!(3x+1)^n$.

5. Find the radius of convergence and the open interval of convergence of $\sum_2^\infty \frac{(-x)^n n^2}{\ln(n)}$.

Power series representation

$$\sum_{0}^{\infty} x^n = \frac{1}{1-x} \quad for \ |x| < 1$$

1. Express $g(x) = \dfrac{1}{1+x}$ as a power series and find its radius of convergence.

2. Express $h(x) = \dfrac{1}{1+5x}$ as a power series and find its radius of convergence.

3. Express $k(x) = \dfrac{1}{5+x}$ as a power series and find its radius of convergence.

 Understand the methods so you can solve similar problems.
Understand the concepts so you can solve unfamiliar problems.

Study the (a) class notes, (b) do the 4th hour problems, (c) study the text examples, (d) do the text exercises and (e) read the next text section.

4. Find a power series representation for $g(x) = \dfrac{x^2}{1+2x}$ and find its radius of convergence.

5. Express *h(x)= ln(1+x)* as a power series and find its radius of convergence.

Study the (a) class notes, (b) do the 4th hour problems, (c) study the text examples, (d) do the text exercises and (e) read the next text section.

Mathematics 126 Fourth 10.6, Power series

Find the radius of convergence and open interval of convergence of the series

1. $\sum_{n=1}^{\infty} \frac{(-3)^n (5x-2)^n}{(n+1)^2}$.

2. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{(2n+1)!}(x-2)^n$.

3. $\sum_{n=1}^{\infty} \left(\frac{(-3x-a)^n}{5^{2n}} \right)$.

4. Find a power series representation and the radius of convergence for
$f(x) = \ln(1-2x)$

$f(x) = \frac{x}{2x+3}$

$g(x) = \frac{x}{5+2x^2}$

$h(x) = x ln(x+1)$

 Understand the methods so you can solve similar problems.
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CAPILANO UNIVERSITY

Study the (a) class notes, (b) do the 4th hour problems, (c) study the text examples, (d) do the text exercises and (e) read the next text section.

10.7 Taylor Series $\sum_{n=0}^{\infty} \frac{f^{(n)}(c)}{n!}(x-c)^n$

To find Taylor series, use a table with columns $n, f^{(n)}(x), f^{(n)}(c), \frac{f^{(n)}(c)}{n!}$.

1. Find the Taylor series for $f(x) = x^4$ centered about $c = 1$.

2. Find the Taylor series for $f(x) = \sin(x)$ centered at $c = \pi/2$.

The **Macluarin series** $\sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n$ is the Taylor series with $c = 0$.

3. Find the Maclaurin series for $f(x) = e^x$ and find its radius of convergence.

 Understand the methods so you can solve similar problems.
Understand the concepts so you can solve unfamiliar problems.

Study the (a) class notes, (b) do the 4th hour problems, (c) study the text examples, (d) do the text exercises and (e) read the next text section.

4. Find the Maclaurin series for $f(x) = \sin(x)$ and find its radius of convergence.

5. Find the Maclaurin series for $f(x) = \cos(x)$ and find its radius of convergence.

6. Find the Maclaurin series for $f(x) = \sin(2x)$ and find its radius of convergence.

Omit Binomial Series and Convergence of Taylor Series on pp 614 to 620.

Evaluating infinite series

1. $Evaluate\ \sum_{k=0}^{\infty} \frac{1}{k!}$

2. Use the infinite series for $\tan^{-1}(x)$ to evaluate $\sum_{0}^{\infty} \frac{(-1)^k}{(2k+1)3^{(2k+1)/2}}$