10.5 The Ratio Test

Omit the Root Test.

Ratio Test: If $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| < 1$ *then* $\sum a_n$ *converges.*

If $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| > 1$ *then* $\sum a_n$ *diverges.*

If $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = 1$ *then the test is inconclusive so use another test.*

1. Determine if the following series converge or diverge.

(a) $\sum_{1}^{\infty}\frac{12^n}{(n+1)5^n}$

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{12^{n+1}/(n+2)5^{n+1}}{12^n/(n+1)5^n}\right|$$

$$= \lim_{n\to\infty}\left|\frac{12}{5}\cdot\frac{n+1}{n+2}\right| = \frac{12}{5} > 1$$, series diverges by the Ratio Test

($\frac{n+1}{n+2} \to 1$)

(b) $\sum n^4 e^{-n}$

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{(n+1)^4 e^{-(n+1)}}{n^4 e^{-n}}\right| = \lim_{n\to\infty}\left(\frac{n+1}{n}\right)^4\frac{e^n}{e^{n+1}}$$

($\frac{n+1}{n} \to 1$)

$$= \frac{1}{e} < 1$$, series converges by the Ratio Test

(c) $\sum_{n=1}^{\infty}\frac{\ln(n)}{n!}$

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{\ln(n+1)/(n+1)!}{\ln(n)/n!}\right| = \lim_{n\to\infty}\left|\frac{\ln(n+1)}{\ln(n)}\cdot\frac{n!}{(n+1)!}\right| = \lim_{n\to\infty}\frac{1}{n+1}$$

($\frac{\ln(n+1)}{\ln(n)} \to 1$)

$$= 0 < 1$$ Series converges by the ratio test.

2. Find all values of τ for which the following series converges: $\sum_{k=1}^{\infty}\frac{\tau^k k}{(k+1)^k}$

$$\frac{n!}{(n+1)!} = \frac{n!}{(n+1)n!} = \frac{1}{n+1}$$

 Understand the methods so you can solve similar problems.
Understand the concepts so you can solve unfamiliar problems.

Study the (a) class notes, (b) do the 4th hour problems, (c) study the text examples, (d) do the text exercises and (e) read the next text section.

10.5

P. 96 #2 $\sum_{1}^{\infty} \frac{T^K K}{(K+1)^K}$

Ratio test

$$\lim_{K\to\infty} \left| \frac{T^{K+1}(K+1)/(K+2)^{K+1}}{T^K \, K/(K+1)^K} \right|$$

$$= \lim_{K\to\infty} \left| \frac{T^{K+1}}{T^K} \cdot \frac{K+1}{K} \cdot \frac{(K+1)^K}{(K+2)^{K+1}} \right|$$

$$= \lim_{K\to\infty} \left| T \cdot 1 \cdot 0 \right| = 0 < 1$$

The series converges for all values of T.