10.4 Alternating Series Test (Leibniz Test)

Omit Absolute and Conditional Convergence. When the text asks if aseries converges absolutely or converges conditionallywe need only show the seriesconverges.

3 **AST or Leibniz Test**: If for some integer M, $a_n > a_{n+1}$ for all $n \geq M$ (1) and $\lim_{n\to\infty} a_n = 0$ (2) then $\sum_1^\infty (-1)^n a_n = S$ converges. Moreover if the series converges to S then

$$|S - S_n| < a_{n+1}$$

1. Estimate the error (to three decimal places) in approximating the sum of the series $\sum_{n=1}^{\infty} \frac{(-1)^n 2^n}{n!}$ by the sum of the first six terms. Is the error less than 0.05? **In your calculator put to find the error, put in u(n) = 2^n/n!, in TABLE read each value as n+1, rather than the n shown at the top of the table.**

$(n+1)! = (n+1)n!$

(1) For $n \geq 2$, $\frac{2^n}{n!} > \frac{2^{n+1}}{(n+1)!}$

$n+1 = \frac{(n+1)!}{n!} > \frac{2^{n+1}}{2^n} = 2$

(2) $\lim_{n\to\infty} \frac{2^n}{n!} = \lim_{n\to\infty} \left(\frac{2\cdot 2}{1\cdot 2}\right) \frac{2\cdot 2\cdot 2\cdots 2}{3\cdot 4\cdots n} = 0$

(3) alternating $\Rightarrow$ AST

So $|S - S_6| < a_7$

$a_7 \approx 0.0254 < 0.05$

YES

2. Determine if the following series converge: (a) $\sum_{n=1}^{\infty} \frac{(-1)^n (n^3-1)}{(n^2+3)(n^2+8)}$

(1) $a_n > a_{n+1}$?

$\frac{n^3-1}{(n^2+3)(n^2+8)} > \frac{(n+1)^3-1}{((n+1)^2+3)((n+1)^2+8)}$

(b) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} \ln(n)}{5\ln(n+1)}$

∞/∞

$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{\ln(n)}{5\ln(n+1)} = \lim_{x\to\infty} \frac{\ln(x)}{5\ln(x+1)} \overset{L'H}{=} \frac{1}{5}\lim_{x\to\infty} \frac{1/x}{1/(x+1)}$

$= \frac{1}{5}\lim_{x\to\infty}\left(\frac{x+1}{x}\right) = \frac{1}{5}\lim_{x\to\infty}\left(1+\frac{1}{x}\right) = \frac{1}{5} \neq 0$ so the series diverges by the Divergence Test.

($\frac{1}{x} \to 0$)

2. Approximate $\sum_{1}^{\infty} \frac{(-1)^n}{3^n n!}$ so that the error is less than 10^{-5}.

$|S - S_n| < a_{n+1} < 10^{-5}$

AST (1) $a_n > a_{n+1}$?

$\frac{1}{3^n n!} > \frac{1}{3^{n+1}(n+1)!}$

$n+1 = \frac{(n+1)!}{n!} > \frac{3^n}{3^{n+1}} > \frac{1}{3}$

true for $n \geq 0$

(2) $\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{1}{3^n n!} = 0$

(3) alternating $\Rightarrow$ converges by the AST

$|S - S_n| < a_{n+1} < 10^{-5}$

$a_6 < 10^{-5}$

$\Rightarrow n+1 = 6, n = 5$

$\sum_1^5 \frac{(-1)^n}{3^n n!} \approx -0.284$

 Understand the methods so you can solve similar problems.
Understand the concepts so you can solve unfamiliar problems.

Study the (a) class notes, (b) do the 4th hour problems, (c) study the text examples, (d) do the text exercises and (e) read the next text section.

3. Determine if the series converges. If the alternating series converges, determine the smallest value of n necessary to estimate the sum and be within 0.01 of the exact sum.

a) $\sum_{n=1}^{\infty} \frac{(-1)^{n-3}(6n^2-2n+3)}{(5+n)(2n+7)}$

$\lim_{n\to\infty} |a_n|$

$= \lim_{n\to\infty} \frac{6n^2-2n+3}{2n^2+17+35} = 3 \neq 0$

So the series diverges by the Divergence Test.

b) $\sum_{1}^{\infty}(-1)^{n+1}5e^{-2n}$

(a) $\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{5}{e^{2n}} = 0$

(b) $a_n > a_{n+1}$?

$\frac{5}{e^{2n}} > \frac{5}{e^{2(n+1)}}$

$\frac{e^{2n+2}}{e^{2n}} > 1$

$e^2 > 1$ ✓

(c) alternating,

Series converges by the AST,

$|S - S_n| < a_{n+1} < 0.01$

$a_4 < 0.01$

$n+1 = 4$

$n = 3$

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10.4

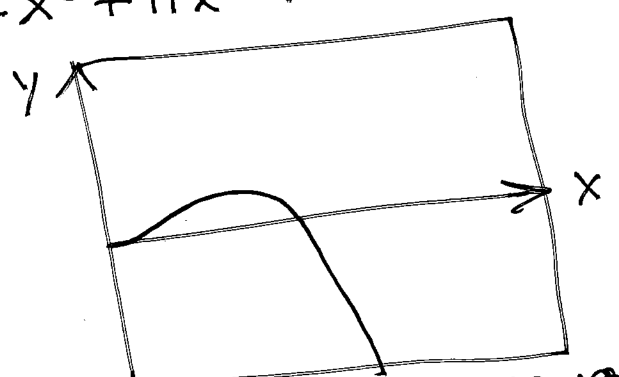
p.93 #2(a) $f(x)=\dfrac{x^3-1}{(x^2+3)(x^2+8)}$

$f(x)=\dfrac{x^3-1}{x^4+11x^2+24}$

$f'(x) = \dfrac{3x^2(x^4+11x^2+24)-(x^3-1)(4x^3+22x)}{(x^4+11x^2+24)^2}$

Numerator of $f'(x) = 3x^6+33x^4+72x^2$
$-(4x^6+22x^4-4x^3-22x)$ $+22x < 0$ for $x \geq 4$

$y_1 = -x^6+11x^4+4x^3+72x^2+22x$



$[0,10]$ by $[-10000,10000]$

(1) ~~so~~ $a_n > a_{n+1}$ for all $n \geq 4$

(2) $\lim_{n\to\infty} a_n = \lim_{n\to\infty} \dfrac{n^3-1}{n^4+11x^2+24}\cdot\dfrac{\frac{1}{n^4}}{\frac{1}{n^4}} = 0$

(3) alternating ~~so~~ by the AST the series conver